Załącznik nr 3

Self-presentation

Contents

1 Introduction

The aim of this self-presentation is to present the scientific achievement submitted for the evaluation in the habilitation process in the context of my academic career.

The text consists of six parts. In the second part I describe my academic education and professional career. In the third part, I present and discuss the series of scientific articles being submitted for evaluation in the habilitation procedure. The fourth part is devoted to discussing some of my remaining research papers. In the fifth part I presented a summary of all my academic achievements. The final part lists the references.

I have characterized my scientific, didactic and organizational achievements in more detail in Appendix 4 to the habilitation application. The list of published scientific works is additionally presented in a bibliometric analysis performed by the Warsaw School of Economics Library, also attached to the application.

2 Education and professional work

In the years 1999-2004 I studied at the Warsaw School of Economics (SGH) in the field of *International Relations*. After presenting the master's thesis entitled *Single-person auction theory*, supervised by prof. Honorata Sosnowska, I graduated with an excellent grade and received my master's degree (magister) on October 21, 2004.

2 Education and professional work

In the years 2002-2004 I studied at the Johannes Gutenberg University in Mainz in the field of *Economics* as part of the *Double Diploma program of the Polish-German Academic Forum*. After presenting two works: a) *Fairness as lack of envy* (written in German), supervised by prof. Georg Tillmann, and b) *Modeling Default Using Copulas* (written in English), supervised by prof. Siegfried Trautmann, I graduated with an excellent grade and received my master's degree in economics (*Diplom Volkswirt*) on October 21, 2004.

In the years 2005-2009, I studied in the international doctoral program at the Department of Economics at the *European University Institute* (EUI) in Florence. Under scientific supervision of prof. Pascal Courty and prof. Fernando Vega-Redondo (auxiliary supervisor) I began doing research in the field of decision theory and behavioral economics.

In 2006, I joined the *European Doctoral Program in Quantitative Economics*, under which I spent the academic year 2007/2008 at the *Bonn Graduate School of Economics* (BGSE) at the University of Bonn, conducting research under the supervision of prof. Paul Heidhues.

In May 2007, I received the title of Master of Research awarded by the Economics Department of the EUI. In September 2008, I conducted the original lecture *Background course in mathematics* and in January 2009 I was an instructor for the tutorial classes for the Micro III lecture, both given for the first-year PhD students in economics at the EUI.

In the years 2007-2010 I participated in a series of prestigious advanced summer schools and scientific workshops, among others *Summer School in Economic Theory* at the Hebrew University of Jerusalem (2007, 2008), *Tinbergen Institute lectures* in Rotterdam (2007, 2009), decision theory workshop at *Centro di Ricerca Matematica di Enrico de Giorgi* in Pisa (2010).

I received the title of *doctor of economics* on January 15, 2010 after presenting the doctoral dissertation entitled *Risk attitudes and measures of value for risky lotteries* and defending the thesis before the doctoral committee consisting of the professors: Pascal Courty (I supervisor) , Fernando Vega-Redondo (II supervisor), Robert Sugden (reviewer, University of East Anglia), Roberto Serrano (reviewer, Brown University). The doctoral thesis was published as the EUI scientific monograph series.

Since October 2010, I have been employed as an adjunct in the Department of Decision Support and Analysis (ZWIAD) of the Institute of Econometrics at the Collegium of Economic Analysis (KAE) of the Warsaw School of Economics, where I conduct scientific research and teach classes. My research focuses on decision theory in the context of risk, uncertainty and time, as well as behavioral game theory. I teach microeconomics, decisions and game theory as well as operations research. I supervise bachelor as well as master students. Most of my lectures are given in English.

During my employment at Warsaw School of Economics I have had several scientific visits, the aim of which was to consult research work or work together on scientific results, including:

a) Study trip (II 2013) to consult my research: University of California San Diego (Mark Machina, Joel Sobel), University of California Los Angeles (Rakesh Sarin), University of California Irvine (Igor Kopylov), Stanford University (Andrzej Skrzypacz), the Rady School of Management (Harry Markowitz), the State University of California in Fullerton (Michael Birnbaum, Allen Parducci),

b) Study trip (IX 2013) to consult my research: University of California San Diego (Mark Machina), Harvard University (Tomasz Strzałecki), Boston University (Jawaad Noor), Massachusetts Institute of Technology (Drazen Prelec),
c) Seminar and joint writing of a scientific article (III 2018): Universitat Pompeu Fabra in Barcelona and Cap Sa Sal (at the invitation of Manel Baucells, professor of the Darden School of Business in Virginia).

In addition to work at SGH, from January 2011 onwards I am also employed as a consultant at the Central Statistical Office in Warsaw (GUS). As part of the work at the Central Statistical Office, I deal with the development of innovative methods of decomposing the following variables: the growth of GDP per capita, the differences and changes in differences in GVA per capita, and the inequality of GDP *per capita* according to Theil's coefficient. These methods are used to identify regional differences.

3 Scientific achievement presented for evaluation

I hereby present the following series of publications under the common title "**Rational and behavioral approach in modelling decisions made under conditions of risk**" as a scientific achievement within the meaning of the Act of 14 March 2003 on academic degrees and academic titles and on degrees and title in the field of art (Article 16, paragraphs 1 and 2), constituting a significant contribution to the development of the discipline of economics in the field of economic sciences.

1) **M. Lewandowski** (2013), "**Risk Attitudes, Buying and Selling Price for a Lottery and Simple Strategies**", *Central European Journal of Economic Modeling and Econometrics* 5 (1): pp. 1-34, **MNiSW[1] = 8**.
2) **M. Lewandowski** (2014), "**Buying and selling price for risky lotteries and Expected Utility theory with gambling wealth**", *Journal of Risk and Uncertainty* 48 (3): pp. 253-283, **IF = 1,125, MNiSW = 35**;
3) **M. Lewandowski** (2017a), "**Prospect Theory versus Expected Utility Theory: Assumptions, predictions, intuition and modeling of risk attitudes**", *Central European Journal of Economic Modeling and Econometrics* 9, pp. 275-321, **MNiSW = 14**;
4) **M. Lewandowski** (2018), "**Complementary symmetry in Cumulative Prospect Theory with random reference**", *Journal of Mathematical Psychology* 82, pp. 52-55, **IF = 2,176, MNiSW = 40**;
5) K. Kontek, **M. Lewandowski** (2018), "**Range-dependent utility**", *Management Science* 64 (6), pp. 2812-2832, **IF = 3.544, MNiSW = 40**;

In the above articles, I consider the following research problems:

i. Explanation of the so-called expected utility paradoxes with particular emphasis on the paradoxes related to the buying and selling prices for risky lotteries, such as: buying/selling price disparity, preference reversal and complementary symmetry;
ii. Determination of the minimal degree of departure from full rationality that is necessary to explain expected utility paradoxes;

[1] MNiSW stands for the Ministry of Science and Higher Education of the Republic of Poland.

iii. The distinction between the expected utility theory on one side and expected utility models on the other in the context of the debate between behaviorists, who criticize the theory of expected utility and postulate its replacement by prospect theory, and rationalists, who refute much of this criticism;
iv. Creating a utility theory and the axiomatic representation of range-dependent preferences that explain the most robust expected utility paradoxes in a unified way that requires less deviation from rationality on the side of decision maker than the previous explanations.

My scientific achievement fits into the broad context of decision modeling in the conditions of objective uncertainty (risk) and a lively and emotionally stirring scientific debate between rationalists, representing the classical and normative approach, and behaviorists, representing the psychological and descriptive approach. Because of the interdisciplinary character of the studied research area, the resulting multiplicity of methodological approaches and lack of consensus regarding the basic assumptions of the theory, in the following part I define the methodology adopted in my research and outline the contentious issues and main positions of either side of the aforementioned debate. Then, in section 3.2, I set out my position, which is a crucial part of my research agenda implemented in the scientific achievement presented for evaluation. It is only in such a broad context that I discuss in section 3.3 the details of my scientific results being part of this achievement. Section 3.4 contains the summary of these results.

3.1 Research area

Theory of decision under objective uncertainty

A choice under conditions of uncertainty is a kind of individual choice, where choice consequences (also called payoffs) are not predetermined, but depend on the resolution of uncertainty, that takes place after the decision has been made. Such a choice is modelled by having consequences depend not only on the choice of an alternative but also on the occurrence of one out of several states of the world (also referred to as states of nature).

There are two types of uncertainty (Machina, Siniscalchi, 2014): subjective, in which the objective probabilities of individual states of nature are unknown or may not even exist, and objective, also known as risk (see Knight, 1921), in which these probabilities exists and are known to the decision maker. Uncertainty is represented by a random variable, called an act[2], which assigns elements from the set of consequences to elements from the set of states of the world. Objective uncertainty occurs when the measurable state space is a probabilistic space. Such uncertainty is represented by the probability distribution, of an act. Such probability distribution is called a lottery or a gamble. An act or a lottery are examples of elements of a choice set. The decision consists in choosing one element from a subset of the choice set. This subset represents the choices available in a given choice situation.

In order for the theory to be testable and falsifiable, it must be based on observable elements. Therefore, it is assumed in the classical decision theory that preferences are directly

[2] Sometimes the term Savage act is used to distinguish it from the Anscombe-Aumann act, in which elements from the set of states are assigned probability distributions of a random variable. See Savage (1954) and Anscombe-Aumann (1963).

revealed via observed choices instead of being based on unobservable elements such as declarations or subjective feelings. The principle stating this is called the revealed preference principle. According to this principle, if the decision-maker chose element f from the subset F of the choice set, then, for each element f' from the set F, the element f is at least as good for him as f'.

The statement "at least as good as" is represented by means of a binary relation on the set of choices and denoted as $\succsim$. If the relation fulfills certain postulates/axioms, then it is called the preference relation. In the theory of decisions under the conditions of objective uncertainty (or risk, in short) it is assumed that decisions, and therefore also preferences, depend only on the probability distribution of payoffs and hence are independent of the states of the world in which payoffs occur. The preference relation is then a subset of the Cartesian product of the set of lotteries with itself.

Decisions under conditions of risk are thus formally modeled as a collection of lotteries and the preference relation defined on this set. Depending on the axioms, that reflect the rules that govern (the descriptive approach) or should govern (the normative approach) choices between lotteries, different theories/models of decision making are distinguished.

The Expected Utility Paradigm

The most widespread and best known is the theory of expected utility (EU theory). As a hypothesis, it was formulated by Bernoulli, D. (1738) as a way of resolving the St. Petersburg paradox posed by Bernoulli, N. (1713). The axiomatic representation of EU preferences was first proposed by von Neumann, Morgenstern (1944). The most compact formulation is that of Fishburn (1970). It is based on three axioms: weak order (the relation is full and transitive), continuity, and independence.

The EU theory was initially conceived as a cardinal way of measuring utility in the context of mixed strategies in zero-sum games, but quickly gained great popularity in economics – it has been adopted in the leading trends of economic modelling as the standard way to model decisions under conditions of risk. Many important hypotheses and models in economics are based on EU theory; examples are numerous across several disciplines of economics: hypotheses of permanent income (Friedmann, 1957) and rational expectations (Lucas, 1972) in macroeconomics; portfolio selection theory (Markowitz, 1952), Capital Asset Pricing Model (Sharpe, 1964) and the efficient market hypothesis (Fama, 1965) in finance; the theory of auctions (Vickrey, 1964) and models of asymmetric information (Akerlof, 1970) in microeconomics to name just a few most important examples.

The popularity of EU theory comes from the fact that it is a normative theory, i.e. one that describes how decisions should be made in a rational way. Axioms of this theory are treated as postulates of rationality in the context of decisions under conditions of risk. People whose behaviors are inconsistent with the axioms of this theory are susceptible to the acceptance of the so-called Dutch books (Yaari, 1985).[3]

[3] A Dutch book is an equivalent of arbitration in decision theory. The Dutch book is a sequence of proposed exchanges that, if made, lead to a certain loss for the acceptor and a certain profit for the proposer, also known as the bookmaker. The decision maker is susceptible to Dutch books if you can construct a Dutch book that he accepts.

The mathematical formalism of EU theory and its convenient properties, such as the linearity of the indifference curves, make the theory particularly easy to handle: the calculations are simple and it is possible to take into account various attitudes towards risk in a parsimonious way. A good illustration of this latter advantage is the fact that scientists almost unanimously accept the definition of risk aversion as one of the commonly occurring attitude towards risk and agree on the method of its measurement proposed by Arrow (1965) and Pratt (1964). At the same time, however, there is no consensus in the scientific community as to the definition and method of measuring risk itself (Aumman, Serrano, 2008).

Rationalists vs. behaviorists debate

The main weakness of the EU models stems from their normative nature and simplicity. Many decisions are not taken in a market-like environment, where consequences of any decision are easily verifiable, thus making it possible for the decision maker to realize his potential mistake and to correct it accordingly. It also turns out that there are factors beyond those included in the EU models that may have a significant impact on real decisions.

Starting from the classic Allais paradox (Allais, 1954) and ending with the Rabin's paradox (Rabin, 2000), over the years a great body of experimental data has accumulated that documents the alleged deviations of the observed behavior from that consistent with EU theory. If a given deviation of such kind is replicated repeatedly in various controlled experimental studies, in which the possible influence of third factors is limited, then such a result is commonly referred to as the EU paradox. It can be said that the behavioral trend in economics is largely based and motivated by the existence, discovery and explanation of the paradoxes of classical normative theory and in particular the EU paradoxes for decisions under risk.

The criticism of the standard approach in economics has triggered the reaction of scientists representing this approach. For simplicity, it can be assumed that nowadays there exist two views: the classical one, also known as normative or rational, and the behavioral one, also called descriptive or psychological. For the sake of simplification, from now on, we shall use the terms: rationalists and behaviorists.

A good illustration of the nature of the scientific debate between these two trends is the following. Based on the impressive amount of EU paradoxes, Rabin, Thaler (2001) declared that "expected utility is an Ex-hypothesis" and that they feel very much "like a customer in a pet shop beating away the dead parrot", alluding to the famous sketch from *Monty Python's Flying Circus*.[4] While postulating the abandonment of EU theory, the authors call for the adoption of prospect theory (Kahnemann, Tversky, 1979, Tversky, Kahnemann, 1992, PT for short) as a theory that solves most of the problems of EU theory.

In one of the articles written in response to the aforementioned criticism of EU theory, Rubinstein (2006), using a fairy tale on the canvas of the biblical story of Adam and Eve in the Garden of Eden, tries to point out the flaws in the argumentation presented by behaviorists: he argues that the possibility of absurd conclusions drawn on the basis of reasonable assumptions

[4] Rabin, Thaler (2001), p. 230. In this sketch, a customer who came with a complaint to a pet shop, tries to convince the seller that the parrot he bought in this store is dead.

(dilemma of absurd conclusions) is an inherent part of every economic model. According to Rubinstein, the model should be treated as a fairy tale rather than a detailed description of reality. And it is the moral that is the most important part of the fairy tale, not the fact that the story does not exactly match reality. Hence, the EU model giving absurd implication in some situations should not be the reason to abandon it. Otherwise, we would have to abandon all models.

The following question seems to lie at the heart of the dispute between behaviorists and rationalists: What qualities should a good model of decision-making have? Behaviorists believe that a good decision model is one that accurately reflects actual decisions, both in the sphere of assumptions, in the proposed mechanism for explaining the decisions as well as in the prediction of the decisions made. In the rationalist approach, the result rather than the assumptions and explanatory mechanism is the key. According to the classic "as if" principle, formulated by Friedman (1953) and later promoted among others by Aumann (1997), a good model should not be judged on the basis of its assumptions, but rather on the basis of the accuracy of its forecasts.

3.2 Research agenda

In a behaviorist dispute with rationalists, I occupy an intermediate position. The arguments of each of the sides that I consider to be relevant are presented below.

Rationalist perspective

The basic argument for adopting a rationalist perspective is stability and coherence of behavior. The possibility of arbitrage makes irrational decisions unstable, i.e. they tend to disappear. This is because decision-makers either correct their decisions after finding out they have made a mistake that cost them their resources, or are unable to make further decisions, e.g. if the wrong decision has led them to bankruptcy.

The role of the normative decision theory is to determine, based on the chosen value system, which decisions are compatible with it and which are not. In particular, the decision-maker sets his/her goal (e.g. maximizing profit) and principles of behaving in certain simple situations in which decisions seem obvious (axioms). The task of the theory is to determine the logical implications of the axioms in the context of the chosen goal, so as to classify all possible decisions into ones that are or are not compatible with them. It is logical, therefore, that decisions belonging to the latter group should be considered a mistake by the decision maker. In order to formally demonstrate the aforementioned implications, decision theorists use what is called a preference representation theorem that shows the equivalence between a given representation (e.g. the EU representation) and the preference axioms (e.g. the EU axioms).

Secondly, a good model should be falsifiable according to the criteria of Popper (1934), i.e. it should clearly separate those choices that are compatible with the model from those that are not. Given an actual choice, it is determined whether the model is compatible with it, confirming its predictive ability, or not, thus falsifying the model. A good model shouldn’t be too flexible so that it will explain any behavior. Such a model is worthless, because on its basis one could not predict what will happen in events and contexts other than those on the basis of which the model was built. The level of generality of the model is closely related to the above argument. A good model should apply in a wide spectrum of decision situations, in particular it should retain predictive power in contexts and decision-making problems beyond those on the basis of which it was build.

Behavioral perspective

The principle of revealed preferences and the resulting dependence of the theory on directly observable elements, is a very effective methodological assumption. This is a self-limitation, that made the level of testability in economics comparable to that of natural sciences. The "as if" principle, postulated by Friedman and Aumann, allows to ignore the simplification and lack of realism of the model if the model gives good predictions. However, there are often situations in which, apart from the prediction itself, we are interested in explaining and understanding behavior.

For example, many decision-making models explain the intersecting sets of possible choices. The Allais paradox can be explained both by PT and regret theory (Bell, 1982, Loomes, Sugden, 1982), and by dozens of other theories and models that have been proposed in the literature. If there are several ways to explain the same phenomenon, then the natural question arises which of these explanations is true.

Decisions are made by people. To understand them, one should not ignore what people are like. In this context, explanations that are consistent with common sense and psychological intuition are naturally preferred. For example, regret theory can explain why people buy lottery coupons (Gollier, C. 2016), but this explanation would ignore the fact that a person who did not buy a coupon cannot feel regret because he often does not know what number he would choose if she played and what number came out in this draw.

Complementarity of both perspectives

Both perspectives, i.e. the normative (as it should be) and the descriptive (as it is) ones, are important and may coexist as complementary approaches. On one hand, the normative aspects should have an impact on people's behavior. For example, if the decision maker may be convinced that his decision was wrong by demonstrating the logical flaw in a sequence of his choices, then the natural implication is that he will avoid making similar decisions in the future.

On the other hand, people's behavior can influence the normative assessment of a given behavior. For example, if in spite of persuasion you cannot convince a reasonable man that his decision was wrong, then maybe your arguments are too weak or the considered model that you have in mind, omits important aspects of the problem. It may happen that the model, and in particular the space of possible choices or states of the world[5], is determined in a way that ignores important aspects of either the problem or its choice alternatives that may affect behavior. Then independence of decisions from these aspects becomes the implicit assumption of the model. Consequently, it may happen, that a given behavior is unreasonable in the original model but becomes rational in a richer model that takes these additional aspects into account.

For example, the silent assumption of many models is independence of decisions from the decision-making context, i.e. from the set of options that are available for choice at a given moment. However, there is a lot of experimental evidence that the decision context impacts many decisions significantly (eg Read D. et al., 1999, Wright, P. 1974). People choose differently under the pressure of time, environment, limited resources, etc. than without such pressure. In the

[5] See e.g. Gilboa (2009), pp. 113-122.

analysis of a model, assumptions that are explicitly defined and those that are hidden in the structure of the problem are equally important (Bastiat, 1850).

A good model is a model that does not hide in its structure assumptions that are not consistent with reality and that may significantly affect the result. The simplifications adopted in the model must therefore be well-founded and in line with the cognitive objective.

Summary of the research agenda

Summing up the research program being carried out in the achievement presented for evaluation, for the observed EU paradoxes I am looking for explanations which, apart from psychological credibility, require the lowest possible departure from the postulates of rationality and thus retain their normative value, and therefore are both simple, general and give strong testable predictions.

Such an approach leads to a particular typology of observed choices according to the degree of their (ir-) rationality: the more one departs from rationality, the more (anomalous) behaviors can be explained. The cost is that in the process the model gradually loses its normative value and predictive power. Therefore, rather than explaining all possible behaviors, the point is to explain those that are robust and persistent.

3.3 Detailed description of the scientific achievement

This section is divided into four interrelated parts that carry out different aspects of the agenda outlined in the previous section. These parts will be discussed in turn:

a) Modeling attitudes towards risk in the models of expected utility;
b) Refutation of the behavioral criticism of expected utility theory;
c) Prospect theory vs. Expected Utility Theory – a critical-comparative approach;
d) Range-dependent utility as an alternative to prospect theory.

Modeling attitudes towards risk in the models of expected utility

This part of the achievement is meant to demonstrate the simplicity and elegance of EU theory. The results of **Lewandowski (2013)** may be applied in testing the decision maker's attitude towards risk. They show the equivalence of directly unobservable elements, i.e. utility and its properties, with observable elements, i.e. the buying and selling prices for risky lotteries and their properties, as well as the so called simple strategies.

Lewandowski (2013) obtains a unified formal characterization of the three most important classes of risk attitudes within EU theory: Constant Absolute Risk Aversion (CARA), Decreasing Absolute Risk Aversion (DARA) and Constant Relative Risk Aversion (CRRA), the latter one being a special case of DARA. In particular, the contribution consisted of:

1) Defining the concept of a simple strategy, that determines whether the decision maker should accept the lottery or not only on the basis of the lottery's own properties and the level of the decision maker's initial wealth. Three basic types of simple strategies are introduced: the wealth-invariant, the scale-invariant and the "wealthier accept more" strategy.
2) Obtaining the representation of the three risk attitude classes using four equivalent methods:
    a. Simple strategies;

b. Properties of the buying and selling price of the lottery;[6]
c. Utility function properties expressed by means of Cauchy functional equations;
d. Utility function properties expressed by means of differential equations.

In relation to Pratt (1964) seminal paper, the characterization of **Lewandowski (2013)** is unified across the three classes and it is extended by adding the properties of buying/selling prices, the formal definition of simple strategies and the analysis based on Cauchy functional equations.

3) Extending the comparative risk aversion theorem by Pratt (1964) by adding the characterization based on buying prices of lotteries as equivalent to that of selling prices.

Refutation of the behavioral criticism of expected utility theory

This is a polemical part, the aim of which is to show that the majority of criticism against EU theory expressed by behaviorists is actually a critique of the standard EU model, in which, apart from a pure mathematical theory, a certain economic interpretation is adopted.

Behaviorists motivate their models in that they explain the EU paradoxes. Rabin, Thaler (2001) postulate replacing the existing EU paradigm with a behavioral theory that would be able to explain the most important EU paradoxes. An important part of their arguments lies in showing that a given preference pattern found in experimental data is inconsistent with EU theory.

In this context Cox, Sadiraj (2006) made an important distinction. EU theory is an abstract mathematical theory based on axioms. To apply this theory in order to model real decisions to be able to test it, it is necessary to supplement this theory with an economic interpretation. A theory supplemented by an interpretation, i.e. assigning economic meaning to the variables and parameters of this theory, is called a model. Thus, there is one EU theory, but many EU models that differ from each other in economic interpretation.

The standard interpretation adopted in most applications of EU theory is the consequentialist interpretation (Rubinstein, 2012). Consequentialism states that:

a) the decision-maker's preferences may be described using a single preference relation $\succsim$ defined over the decision maker's total wealth levels[7], also referred to as terminal or lifetime wealth levels, and
b) preferences over changes of wealth starting from wealth level W, denoted as $\succsim_W$, can be derived from this single preference relation as follows: $P' \succsim_W Q' \iff P \succsim Q$, where P, Q are lotteries defined on total wealth levels, P', Q' are lotteries defined on wealth changes and the following holds: $P'(x) = P(W + x), Q'(x) = Q(W + x)$, for each x belonging to the set of lottery payoffs defined as wealth changes relative to the initial wealth level of the decision maker.

EU theory together with the consequentialist interpretation is called the standard EU model.

Lewandowski (2014) is one in a series of articles (including Cox, Sadiraj, 2006, Palacios-Huerta, Serrano, 2006, Rubinstein, 2006, 2012, Foster, Hart, 2009), demonstrating that some

[6] Buying and selling prices of lotteries were first introduced by Luce, Raiffa (1987).
[7] More specifically: lotteries, whose payoffs are levels of total lifetime wealth.

experimental data that allegedly falsify EU theory, in fact falsifies the standard EU model, and in particular its consequentialist interpretation. This kind of argument significantly weakens the justification for Rabin, Thaler (2001) postulate to abandon EU theory as a good descriptive model of choice under risk and to replace it with PT.

In this context **Lewandowski (2014)** explores the possibility of explaining some of the EU paradoxes in – other than the standard one – EU models. Two models are analyzed, in which the consequentialist interpretation has been replaced other economic interpretations:

1. *The EU model of gambling wealth* is based on the concept of mental accounting (Thaler, 1985), in which the decision maker divides the total assets into separate mental budgets that are not exchangeable. Rationality demands are restricted to hold within each single budget instead of holding across all budgets. For example, the same person may avoid risk in his retirement plan and at the same time take risks within the gambling budget.
2. *The EU model of wealth changes*, also called the EU of income model.[8] Instead of wealth levels, lotteries are defined on changes in wealth level. In this model, the utility function is defined on nominal gains and losses with respect to to the *status quo*, i.e. the level of current wealth. An important element of this model is loss aversion, which may be expressed by the following statement: "Losses loom greater than gains" (Kahnemann, Tversky, 1979, p. 279).

Whereas, in other articles of the above-mentioned series, only the Rabin (2000) paradox was analyzed, **Lewandowski (2014)** also analyzes other important EU paradoxes, such as buying/selling price disparity and preference reversal.

In the buying price elicitation task, a subject is asked[9] at what maximum price he is willing to buy the lottery. In the selling price elicitation task, the question concerns the minimum price at which he is willing to give up the lottery. The buying price is also called the willingness to pay (WTP), whereas the selling price is also referred to as the willingness to accept (WTA). Form now on these concepts are used exchangeably.

Starting from Knetsch, Sinden (1984) and Thaler (1980), many experimental articles documented the WTA and WTP values for different types of goods. The results of these works indicate that for a given decision maker the selling price is usually much higher than the buying price. This is true for many types of goods, including risky lotteries. Horowitz and McConnell (2002) document and analyze the results from a large number of experiments and obtain average values of the WTA/WTP coefficient for various goods. The average coefficient for lotteries is 2.10.

In the classical utility model (in the case of decision-making under risk it is the standard EU model), the existence of such a large gap is impossible. This is because in the EU models based on wealth levels (total, gambling, etc.), the buying and selling price of a given lottery may differ only due to wealth effects. Wealth effects arise because while the decision-maker initially does not own the object when buying, he does own it when selling. This asymmetry in the initial property

[8] This model is a special case of the Reference Dependent Subjective Expected Utility (see Sugden, 2003). The reference point in this model is the level of initial assets (*status quo*).

[9] The specific procedure for eliciting prices is based on the mechanism of Becker, DeGroot, Marschak (1964) or on the multiple price list (Kahnemann, Knetsch, Thaler, 1990).

rights in the two cases may be the reason for the wealth effect if the object being bought or sold has significant value for the decision-maker relative to his initial wealth level. In the standard EU model, initial wealth is the decision maker's total wealth, which typically by far exceeds the value of the lottery prizes offered in experiments. In this case wealth effects are negligible, as is the disparity between the buying and selling prices of lotteries.

The situation is different in the case of the EU model of gambling wealth. Lottery prizes may well exceed the level of assets that the decision maker allocates to gambling – after all the essence of gambling is the possibility of earning a much larger sum than that which a gambler enters with. In this model, therefore, a large wealth effect may arise and, consequently, a large disparity between the buying and the selling price of a lottery may occur.

While the main mechanism explaining EU paradoxes in the EU model of gambling wealth are the nonnegligible wealth effects, it is the dependence on the status quo together with loss aversion in the case of the EU model of wealth changes. According to this second explanation, the decision-maker's desire to have a given good (in this case a lottery) depends on whether he already owns it or not. In the first case, the decision-maker may be reluctant to give up the good, because she is averse to losses. If, on the other hand, the decision-maker initially does not own a given good, she values it lower, because its lack is not perceived as a loss. **Lewandowski (2014)** shows that in the EU model of wealth changes, the existence of a large gap between the buying and the selling price is equivalent to the existence of aversion to losses. This result is independent of the assumed definition of loss aversion, among the various ones existing in the literature.

Rabin's paradox is based on the calibration theorem (Rabin, 2000). The theorem states that if an EU decision maker rejects an even chance of receiving $110 or losing $100 at any wealth level, he will also reject an even chance of losing $1000 or getting an arbitrarily large amount. Rabin argues that while the premise seems to be reasonable, the conclusion certainly isn't.

Lewandowski (2014) demonstrates that in the EU model of gambling wealth, the above implication is not at all paradoxical. The fact that people usually reject the first lottery (the fact commonly observed in experiments), does not mean that they would do so at any wealth level. If, in accordance with the concept of mental accounting, dealing with such lotteries belongs to the decision maker's gambling budget, the fact that people usually reject such a lottery may mean that their gambling wealth is too small for the lottery to be considered attractive. If my gambling wealth is less than $100 dollars, then accepting the lottery means 50% chance of resetting my gambling budget. However, for a professional casino player whose gambling wealth is greater than for the average person, such a lottery is not unattractive at all – its expected payoff is +$5.

Apart from the observed preference patterns that are not consistent with the standard EU model but become consistent with the EU model of gambling wealth, there are patterns that remain inconsistent with the latter model. One such phenomenon is the preference reversal paradox (Lichtenstein, Slovic, 1971, Lindman, 1971, Lichtenstein, Slovic, 1973, Grether, Plott, 1979). In this paradox there are two binary lotteries, each having a single nonzero prize. In a lottery called the $ bet, a non-zero payoff is high, but the probability of winning it is low, while in a lottery called the P-bet, a non-zero payoff is low, but the probability of winning it is high. The paradox is that people often assign a higher certainty equivalent (or selling price) to the $ bet and at the same time express preference for the P-bet in a direct choice between the two lotteries.

This paradox is all the more surprising because apart from explanations based on the violation of procedural invariance, within the class of models based on asset integration such preference pattern is a serious violation of transitivity.[10] Such a violation is the basis for constructing the so-called money pump, i.e. the crudest and thus the most effective Dutch book to pump out money from the decision maker (Loomes, Sugden, 1983).

As such, this paradox cannot be explained within the EU model of gambling wealth, which excludes arbitrage. Nevertheless, **Lewandowski (2014)** shows that under this model, it is possible to explain a similar phenomenon called the "preference reversal B", which is the same as the original reversal except for buying prices replacing selling prices. Such a reversal is possible as it does not necessitate violating the no-arbitrage principle.

In the EU model of wealth changes, on the other hand, the same EU axioms take on a new meaning, as they are assumed to hold for lotteries defined on changes of wealth rather than on wealth levels. This opens the possibility for arbitrage. **Lewandowski (2014)** demonstrates that as a consequence the preference reversal paradox in its original form may be accommodated in such a model and the explanation is based on the existence of loss aversion in its most general form.[11]

The two models analyzed in **Lewandowski (2014)** can be ranked on the scale of departure from the full rationality defined by the standard EU model. Although EU axioms are met in both models, they have different meaning in each of them. The crucial feature that distinguishes the two models is asset integration. In all EU models that are based on wealth (either total wealth or gambling wealth), new assets are integrated with the existing ones. Such integration does not hold in the reference dependent EU models in general and in the EU model of wealth changes in particular. This is important because, as demonstrated by **Lewandowski (2014)**, this lack of integration results in susceptibility to arbitration. This means that the level of rationality loss in the EU model of wealth changes is much larger than in the case of the EU model of gambling wealth.

This distinction is not fully realized in the literature. For example, Yaari (1985) distinguishes four different types of Dutch books. His Dutch book number 4 is aimed to show that the EU maximizers may also be susceptible to arbitrage. In light of **Lewandowski (2014)** his argument may be clarified and put in the right perspective. Dutch book number 4 is possible, but only in the EU model of wealth changes. In all EU models that assume asset integration over the relevant domain of choices, this Dutch book is impossible to occur. This result is an important step on the way to completing the proof of the no-Dutch book theorem for decision-making under risk.[12] Such

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[10] There are also violations of transitivity, which are based on the fact that people are not able to order alternatives in a strict manner if the difference between them is too small. This may lead to intransitivity of indifference (Luce, 1956). However, on the basis of such violations it is difficult to construct a "money pump" in which the decision maker would lose a significant amount of money.

[11] I.e. the decision-maker's unwillingness to accept binary lotteries, in which with equal probability you either gain or lose a given amount of money.

[12] This theorem, still formally unproven, can be characterized as follows: a) If preferences violate any of the EU axioms, then it is possible to construct a Dutch book that will be accepted, b) If one can construct a Dutch book, then the preferences violate at least one of the EU axioms. Part a) of this theorem was first demonstrated by Yaari (1985). The only known counter-example to part b is given in the same article – Dutch book

theorem is crucial in justifying why EU models with asset integration are considered a standard of rationality, with the full level of rationality being achieved in the standard EU model based.

Prospect Theory vs Expected Utility Theory – a critical-comparative approach

The main research question here is: In what ways is PT better or worse than the EU theory? Does the cost-benefit analysis justify the postulated replacement of EU theory with PT?

Although the rationality in the reference dependent EU models in general and the EU model of wealth changes in particular is lower than in the EU model of gambling wealth, in both these models it is significantly higher than in the PT, where apart from reference dependence and loss aversion, a non-linear weighing of probabilities is additionally added.[13] This extra element introduces major departure from EU theory itself, because it not only changes the interpretation leaving the axioms the same, but it also violates independence, the main EU axiom. Independence is key in getting the EU representation, because it plays major role in proving that the indifference curves[14] in the EU theory are straight lines, parallel to each other. This is a crucial property of all EU models, which is expressed in the fact that the expected utility functional is linear in payoff probabilities. Violation of linearity therefore introduces a very serious departure from rationality.

Lewandowski (2014) showed that many important classic EU paradoxes can be explained without the need to introduce this additional source of irrationality. It is also important that the proposed explanations do not require changing the reference point (reframing). The reference point is assumed to be the level of decision maker's initial wealth, i.e. the *status quo*. However, if this additional degree of freedom (possibility of reframing) is added, as is also the case in PT, then on top of explaining the paradoxes analyzed in **Lewandowski (2014)** it is also possible to explain the classic Allais paradox – see Schneider, Day (2016) who use a maximin rule for establishing the reference point given a decision context. The above arguments indicate that reference dependence coupled with loss aversion is sufficient to explain the main EU paradoxes and, hence in accordance with the Ockham razor principle, non-linear weighing of probabilities seems unnecessary.

Lewandowski (2017a) is a review article. Its main purpose is to present prospect theory in a critical-comparative manner in the context of the debate between rationalists and behaviorists. The following questions are addressed:

a) Which assumptions (those hidden and those formulated explicitly) of classical theory are questioned in PT?
b) How PT explains experimental data, while striving to find the right balance between the basic rationality postulates of EU (e.g. monotonicity with respect to the first-

number 4. In the light of the results given in Lewandowski (2014) and Lewandowski (2017b), this counter-example is valid only in the EU models without asset integration. In order to complete the proof, it is necessary to show that there is no other counter-example to part b.

[13] In the original PT probabilities of different payoffs are each weighted separately. In cumulative PT probabilities of getting at least a given level of payoff are weighted.

[14] The indifference curve is a hyperplane characterized by the same Expected Utility value in the set of probability simplex representing finitely supported lotteries.

order stochastic domination), psychological credibility and mathematical elegance?

c) How attitudes towards risk are modeled in PT? In particular, prospect stochastic dominance and the three-pillar structure of risk-attitude modeling in PT were discussed, including: non-additive decision weights with lower and upper subadditivity and their relation to the concepts of pessimism and optimism, as well as preferences divided into preferences within and between the gain and the loss areas (corresponding to the notions of basic utility and loss aversion);

d) What are the most important applications of PT?

The main argument of behaviorists in favor of PT is its descriptive accuracy. However, it turns out that this accuracy is far from optimal in some simple choice situations. **Lewandowski (2018)** analyzes the property of complementary symmetry that holds in PT. This property has been introduced by Birnbaum and Zimmermann (1998) and can be characterized as follows. Let $g = (x, p; y)$ and $g' = (x, 1-p; y)$ be two binary lotteries, where x, y are monetary payoffs such that $x > y$ and $p \in (0,1)$ is a probability of getting payoff x in lottery g or payoff y in lottery g'. Complementary symmetry states that the sum of the buying price of g, denoted as $b(g)$, and the selling price of g', denoted by $s(g')$, is equal to the sum of its payoffs, i.e: $x + y$.

Experimental data (Birnbaum and Sutton, 1992, Birnbaum, Yeary, Luce and Zhao, 2016 and Birnbaum and Zimmermann, 1998) does not confirm this property. In the experiments, buying and selling prices of each lottery in a series of lotteries were elicited for each subject from a group of subjects. The lotteries considered were of the type g, g' as above, with payoffs x, y varying across different lotteries such that the amount $x + y$ was held fixed. It was found that the sum of the median $b(g)$ and the median $s(g')$ values is not constant and depends on the range of payoffs, i.e. on the value of $x - y$. This sum lies always below the value of $x + y$ and decreases as the range increases. For example, Birnbaum and Sutton (1992) showed that the average buying and selling price of the lottery (60 USD, 0.5, 48 USD) are USD 50 and 54 USD, respectively, and therefore their sum is 104 USD. However, the average buying and selling price of the lottery (96, 0.5, 12 USD) are 25 and 50 USD respectively, and therefore their sum is only 75 USD.

The decision maker's situation in the buying task is different than in the selling task. While in the former he contemplates exchanging a sure amount of money for a risky lottery, in the latter the situation is reversed. To model this kind of asymmetry between the buying and selling task within Cumulative PT, Birnbaum and Zimmermann (1998) (Appendix B) proposed two models. In Model No. 1, the utility of the lottery is compared to the utility of the price obtained for the lottery when selling or paid for the lottery when buying. This model is an extension of a similar model that has been proposed for consumer goods (Tversky, Kahneman, 1991). In Model No. 2, lottery payoffs are integrated with the price: the price serves as a reference point for the evaluation of the lottery upon buying, while the lottery serves as a (random) reference point for the evaluation of the price upon selling.

Birnbaum and Zimmermann (1998) identified the key implications of each of the two models and showed that they are incompatible with experimental data suggesting that the range of lottery payoffs, i.e. $|x - y|$, plays an important role in explaining the value of elicited buying and selling prices (see for example Birnbaum and Beeghley, 1997, Birnbaum and Stegner, 1979, Birnbaum and Sutton, 1992). In the case of Model No. 2, the questionable property is complementary symmetry, whereas in the case of Model No. 1 it is the property of constant ratio of selling to buying price of a lottery.

The aim of **Lewandowski (2018)** is to show whether these questionable properties are transferred to the case in which some of the strong assumptions of the parametric cumulative PT model adopted by Birnbaum and Zimmermann (1998) will be relaxed. The main focus was on Model No. 2 (and thus on complementary symmetry), because the main idea of this model, i.e. the integration of lottery payoffs with prices, became a standard in later approaches (see e.g. Luce, 1991). In particular this idea was adopted in the Third Generation PT (Schmidt, Starmer and Sugden, 2008). The appendix of **Lewandowski (2018)** contains the analysis of the less popular Model No. 1 and its implication of a constant ratio of selling to buying price.

Birnbaum et al. (2016) and Birnbaum and Zimmermann (1998) showed that in model No. 2 complementary symmetry occurs regardless of the form of the probability weighing function for gains and losses, if the utility function for gains and losses has the following form: for $\alpha > 0$, it holds: $u(x) = x^{\alpha}$, for $x \geq 0$ and $u(x) = -\lambda(-x)^{\alpha}$, for $x < 0$. The main result of **Lewandowski (2018)** is the demonstration that complementary symmetry also occurs in a much more general model, i.e. for each strictly increasing and continuous utility function, which fulfills the condition $u(0) = 0$. In particular, this property occurs regardless of whether the following elements are or are not included in the model: loss aversion, reflection effect and preference homogeneity (i.e. separate power utility function for gains and for losses).

Range-dependent utility as an alternative for prospect theory

This part is the culmination of the research agenda outlined in part 3.2 above. Previous sections discuss the work aimed at demonstrating that EU Theory is not an "Ex-hypothesis" and that the postulates of replacing it by PT are premature. The arguments presented there, however, are mostly of polemic nature and thus are not entirely constructive. If, as argued, PT is not a good replacement for the standard EU model, the natural question arises: is there a good model that would be suitable for such replacement?

According to me, the answer to the above question is affirmative. **Kontek, Lewandowski (2018)** propose a new theory of decision-making under risk, called range-dependent utility theory (RngDU). In a letter informing us – the authors – of the acceptance of the article for publication after the lengthy process of reviews and corrections, the editor of the Decision Analysis Department of *Management Science* Manel Baucells wrote: "Congratulations for a novel model to account for risk preferences that is thought provoking and compatible with much experimental evidence".

After the publication of the article, Manuel Baucell, Krzysztof Kontek and I have established fruitful scientific collaboration with the goal to extend our range-dependent utility theory to the time domain, thus creating an integrative behavioral model for general uncertain cash-flows that jointly accounts for risk and time paradoxes. The first steps in this direction have been made – see, for example, working paper **Baucells, Kontek, Lewandowski (2018)**, discussed later.

The RngDU theory is an alternative to EU models and PT. It is based on range principle introduced by Parducci (1965) in his Range-Frequency Theory – the well-known theory of psychophysical judgment. The range principle states that people evaluate a given psychophysical stimulus relative to the largest and the smallest of all stimuli present in a given situation. For example, on tropical islands, where the temperature varies in the range of 27-32 degrees Celsius throughout the whole year, the natives are sensitive to temperature changes that would be almost imperceptible for Polish residents; therefore, they complain about heat if the temperature exceeds 30°C or cold if it does not exceed 28°C.

Such range effects may be found in many contexts outside psychophysical judgments. For example, they are an important factor explaining consumer behavior on the market - Cialdini (1993) gives many examples. **Kontek, Lewandowski (2018)** were, however, the first who applied these effects to explain choices under risk.

The main contribution of **Kontek, Lewandowski (2018)** is as follows. First, this paper introduces and axiomatizes range-dependent utility as a new conceptual framework for decision making under risk (henceforth referred to as RngDU theory). It is a simple and well-defined generalization of expected utility theory in which utility depends on the range of lottery outcomes. Second, a special case of this framework is proposed for prediction. It is based on applying a single utility function (decision utility) to every normalized lottery range. The resultant decision utility model (henceforth referred to as the RngDU model) predicts well-known expected utility paradoxes without recourse to probability weighting. Necessary and sufficient conditions for the model to satisfy monotonicity with respect to first-order stochastic dominance are identified. The typical decision utility function, which is confirmed by both experimental data and normative considerations, is S shaped.

In comparison to prospect theory, both the RngDU theory as well as the RngDU model have a number of advantages that can be divided according to the important criteria for model evaluation, drawn up in my research program outlined in section 3.2:

In this case, RngDU model is equally supported as rank dependence. Descriptive predictions of the two concepts differ in the case of multioutcome lotteries, with a lot of evidence supporting range dependence as discussed in Section 6.2.

Descriptive accuracy: Most of the existing experimental evidence in decision making under risk involves binary lotteries, i.e. lotteries in which there are two payments with non-zero probabilities. In this case, the RngDU model is observationally equivalent to the rank dependent expected utility (Quiggin, 1982, Yaari, 1987). This means that all data that confirm the concept of cumulative probability weighing, which is the crucial part of CPT (Tversky, Kahnemann, 1992), at the same time confirm to the same extent the concept of RngDU, in which there is probability weighting of any kind. Therefore, it is wrong to claim, as many behaviorists do[15], that a certain experimental data concerning binary lotteries is evidence of nonlinear weighting of probabilities. Predictions of rank-dependent utility differ from those of range-dependent utility in the case of multiple outcome lotteries. This allows the comparison of descriptive accuracy of both models. In this case, **Kontek, Lewandowski (2018)** cite a number of experimental studies and invoke many arguments indicating that the range dependent utility model has a clear advantage over the rank dependent utility model.

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[15] For example, Kahneman, Tversky (1979, p. 283) invoke the so-called Zeckhauser paradox involving only binary lotteries to confirm the hypothesis of nonlinear weighting of probabilities.

Psychological credibility: Concepts of rank and range dependence offer, however, an entirely different psychological explanation of the same empirical evidence. Unlike its mathematical elegance[16], psychological plausibility of cumulative probability weighting is disputable (Birnbaum 2004) as it is not based on any psychologically plausible arguments. Range dependence, on the other hand, seems more natural—it is based on the well-established psychological theory of Parducci (1964).

Normative arguments: EU paradoxes may be explained using different models. The aim of the RngDU model is to simultaneously explain the most common EU paradoxes in a way that requires the smallest departure from the full rationality of the EU standard model. Therefore, the RngDU model retains linearity in probabilities and converges to the standard EU model as decision makers enrich the support of the lotteries and make the ranges wider.

Predictive power: the RngDU model gives strong testable predictions because it is falsifiable and parsimonious: similar as in the standard EU model, it has only one degree of freedom – the shape of the decision utility function. In CPT, by comparison, there are many more, i.e. the shape of: the value function, the probability weighting function as well as the location of a reference point. The disadvantage of the RngDU model, however, is the lack of loss aversion and reflection effect for gains and losses. Such a generalization is developed in my collaboration with Manel Baucells from the Darden School of Business at the University of Virginia and with Krzysztof Kontek from Warsaw School of Economics (see working paper **Baucells, Kontek, Lewandowski, 2018**)

My contribution to **Kontek, Lewandowski (2018)**, consists of:

a) I axiomatized the RngDU theory and the RngDU model i.e. proposed the axioms, demonstrated their consequences and intuition, formulated and proved two representation theorems (Theorem 1-2)
b) I formulated and proved the theorem showing the discontinuity of the model wrt. the convergence of probability distributions and characterized the necessary and sufficient conditions for the conformity of the RngDU model with the first-order stochastic dominance (Theorem 3),
c) I derived the necessary and sufficient conditions needed to explain the most important EU paradoxes by the RngDU model,
d) I prepared discussions on: continuity and monotonicity in general and in the Marschak-Machina triangle, explanation of the EU paradoxes, advantages of the RngDU model and RngDU theory, and their comparison with related models and theories existing in the literature,
e) I proposed and discussed the motivativating example on the Powerball lottery.

3.4 Summary of the most important contributions

Summing up, in the series of articles submitted for the evaluation, I consider the following contribution to be particularly important:

[16] The idea is taken from Quiggin (1982). It was also applied in the Choquet Expected Utility (Schmeidler, 1986, 1989) in the richer context of subjective uncertainty.

1) I formulated and proved the axiomatic representation theorems of range-dependent utility theory and the predictive range-dependent utility model. I characterized formal properties of the model such as monotonicity and continuity, and defined the necessary conditions to explain within the model the most important expected utility paradoxes;
2) I demonstrated that the most important among the so-called expected utility paradoxes can also be explained within the framework of expected utility theory, if the implicit interpretation adopted in the standard model is replaced by an interpretation based on mental accounting or reference dependence and loss aversion.
3) I demonstrated that the existence of a large gap between buying and selling prices of lotteries is possible within the expected utility model of gambling wealth and that wealth effects may explain a significant part of this gap.
4) I made a typology of expected utility models and prospect theory in terms of their degree of deviation from the full rationality of the standard model of expected utility.
5) I demonstrated that under the expected utility models, the Dutch Book number 4 introduced by Yaari (1985) is only possible in a reference dependent model, while it is not possible in models based on wealth levels. This is an important contribution to completing the full proof the No Dutch book theorem for decisions under risk.
6) I created the formalism of buying and selling prices of lotteries within the models of expected utility. Using this formalism, I extended characterization results of the most important classes of risk attitudes, linking attitudes towards risk on the part of decision maker with the riskiness of the lottery, and offered an explanation of those expected utility paradoxes that involve buying and selling prices.
7) I performed a critical-comparative analysis of prospect theory in relation to expected utility theory and range-dependent utility theory based on the following criteria: predictive power, the embedded rationality of decision maker, psychological intuition, descriptive accuracy, the ease of modelling attitudes towards risk, the level of generality and simplicity of the model.
8) I demonstrated that the property of complementary symmetry, violated in all known experiments that test it, occur in the most general formulation of prospect theory with a random reference point. This result further weakens the role of prospect theory as a good replacement alternative for expected utility models.
9) I contributed to demonstrating that expected utility theory is not an "Ex-hypothesis" but rather that “the expected utility parrot may well be saying that *the report of my death was an exaggeration*” (Palacios-Huerta, Serrano, 2006, p. 258).

4 Other scientific achievements

In addition to the articles that I point out as my achievement, there are a number of other scientific papers that I wrote. A large part of them is in the form of working papers, which will soon be submitted in highly respected scientific journals with impact factor.

As part of continuing work on range-dependent utility theory, **Baucells, Kontek and Lewandowski (2018)** generalize this theory by extending the domain of its application to decisions concerning uncertain cash flows over time. The result is a behavioral model, called the range and sign dependent utility for risk and time, that offers a unified explanation of the EU paradoxes for risky lotteries as well as the paradoxes of the discounted utility model for cash flows over time. For lotteries played today, the model can be seen as an extension of the original prospect theory based on range effects instead of rank effects. In the case of delayed payments, the model is consistent with hyperbolic discounting. In the most general case of uncertain cash flows, it adopts an original shape.

The model uses an endogenous framing rule, which sets the range and reference point, and three functions: a value function that takes into account loss aversion, the S-shaped range distortion function and the subjective survival function. Range and sign dependent utility jointly explains among others: the classic Allais paradoxes, Samuelson's paradox for risk and time, the phenomenon of preference reversal and, for time, decreasing impatience and the magnitude effects.

My contribution to this article is to propose the shape of the model in its most general form, i.e. for uncertain cash flows and to develop conditions that must be met in order for the model to explain paradoxes for risk and time domains. I also stated the representation theorem based on axioms of range and sign dependent preferences.

As a continuation of the part of my research agenda on refuting the behavioral criticism of expected utility theory and critically evaluating prospect theory, **Lewandowski (2017b)** demonstrates that the interpretation based of reference dependence and loss aversion coupled with EU theory is sufficient to explain many important EU paradoxes and that it is not necessary to introduce a non-linear weighting of probabilities. Sugden (2003) was the first to analyze in a formalized way the reference dependent EU model in a richer context of subjective uncertainty. Based on the formalism of this article, **Lewandowski (2017b)** shows that the following EU paradoxes may be explained within the reference dependent EU model: the WTA / WTP gap, the phenomenon of reversal of preferences, complementary symmetry, homogeneity of preferences, aversion to losses, the rebound effect and coexistence of insurance and gambling.

As a continuation of the part of my research agenda demonstrating the elegance of modeling risk attitudes in expected utility theory, **Lewandowski (2010)** analyzes the operational measure of riskiness, that was introduced by Foster, Hart (2009). This is a measure that is based on models of EU of wealth. For a given lottery, its riskiness is the minimum level of initial wealth that a decision maker must have in order to accept the lottery safely.

Foster, Hart (2009) showed that in an infinite series of sequentially offered gambles the strategy of never accepting a gamble if its riskiness measure falls below the decision maker’s current wealth, guarantees that the decision maker will never go bankrupt (the probability his wealth converges to zero is zero). Of all the strategies that provide such a guarantee, the EU decision maker whose utility function assumes the logarithmic form, is the least conservative – accepts the largest number of lotteries.

Lewandowski (2010) generalizes an operational measure of riskiness in two aspects: first, he obtained the necessary and sufficient conditions for the existence of a generalized measure of riskiness that instead of the logarithmic class is based on either of the two more general classes: the CRRA class, or even more general DARA class.

The second generalization of the risk measure proposed by **Lewandowski (2010)** consists in defining a measure of riskiness for lotteries with prices. The riskiness of the lottery itself may not exist – it is well defined only for a lottery with positive expectation and some negative elements in its finite support. After taking into account the lottery price in the range of possible values of buying or selling prices of this lottery when wealth level varies, the riskiness measure is always well-defined. Depending on the amount of the price, the order of lotteries based on their riskiness is defined and its properties are analyzed – a number of claims concerning properties of the buying and selling prices in relation to the measure of riskiness of the lottery are stated and proved.

In my research I also dealt with decisions in which instead of uncertainty concerning the occurrence of (external) states of the world, there is uncertainty concerning the (internal) decision maker's preferences. Most people consider health (quality and duration of life) as important but since we rarely choose between health states, our preferences are often not well-formed; moreover, the quality of life is frequently defined using imprecise terms (e.g. moderate difficulties doing usual activities). **Jakubczyk, Kamiński, Lewandowski (2018)** propose to model preferences towards health states (precisely: disutilities of worsening health dimensions in the EQ-5D-5L descriptive system) as fuzzy: each worsening is assigned an interval instead of a crisp number. We elicit such preferences with discrete choice experiment (DCE) data, using a maximum likelihood approach and bootstrapping to assess the estimation error.

My contribution in the publication consisted in comparing the approach proposed in the paper to other approaches to model imprecise preferences existing in the literature (excerpts from the chapter Discussion on pages 143-145 in the original publication) and the co-editing and acceptance of the final version of the text.

In addition to the theory of individual decisions, in my research I also deal with interactive decisions – games. Goeree, Holt (2001) present data from experiments on a series of two-person games that are played once. These games include standard game types: static and dynamic games with complete and incomplete information. For every game there is a "Treasure" treatment means such a selection of the game parameters that the observed behavior in these games is consistent with the Nash equilibrium strategies or the equivalent (Subgame Perfect Nash Qquilibrium in dynamic games and Bayesian Perfect Equilibrium in games with incomplete information). For each "treasure", the key payoff parameter has been changed in a way that does not change the equilibrium predictions, but dramatically changes the observed behavior, leading to discrepancies between predictions of game theory and real behavior. These contradictions are in line with economic intuition, but there is no satisfactory theoretical explanation for them.

Lewandowski (2016) proposes an explanation of the occurrence of these discrepancies based on the so-called strategies of minimax regret. The assumption underlying the standard models of game theory is the assumption of common knowledge of rationality (Pearce, 1984, Bernheim, 1984). It states that every player is rational (i.e. maximizes Expected Utility of mixed strategies), and every player knows that the other players are rational. Each player also knows that each of the other players knows that other players are rational and so on in an infinite sequence of recursively built knowledge. In experiments, the assumption is usually not met because players usually have doubts about the rationality of other players. In games, there are situations when playing an equilibrium strategy is risky, i.e. if I play an equilibrium strategy and my opponent does not, I will get a much lower payoff than if I did not play the equilibrium strategy just like my opponent. In such situations, players may treat the game as a decision under the usual – not a strategic – uncertainty, i.e. treat the opponent as if it was impossible to predict his behavior. In fact, a good strategy is to minimize the maximum regret.

Lewandowski (2016) shows that minimax strategies explain deviations from the equilibrium strategy in the analyzed games. This working paper is part of a larger project consisting in testing the predictions of game theory and verifying proposed explanations for these deviations. The research project I am a leader of is implemented as part of the statutory research of the Collegium of Economic Analysis at SGH. In January 2019, it is planned to release a mobile application created within the project. This application will be available for free in the Google Play online store. The goal of this application is to collect data on static games with complete information and test the hypothesis of minimax regret strategies.

5 Summary of scientific achievements

At the time of submitting the application, the list of my scientific publications (after being awarded the *Doctor of Economics* title) includes:

- 3 articles published in magazines having impact factor (IF) from list A of the MNiSW,
- 3 articles published in magazines from the B list of the MNiSW,
- 10 working papers,
- 2 chapters published in collective monographs,
- 1 scientific monograph,
- 1 expert opinion and 2 reports on research and development projects,
- 7 chapters in reports,
- 1 report in a journal awarded the MNiSW points.

The total IF for the publications mentioned above is 6.845, and the total value of the MNiSW points is 180. The total number of citations of my works (according to the bibliometric analysis, excluding the date of publication) is:[17]

- 44 (according to Google Scholar),
- 6 (according to the Web of Science),
- 5 (according to Scopus).

These citations translate into the Hirsch index equal to:

- 4 (according to Google Scholar),
- 2 (according to Web of Science),
- 1 (according to Scopus).

I presented the results of my scientific research at many conferences both at home and abroad:

[17] Explanation for a large difference in the number of citations by various sources: The presented articles are theoretical. Such articles are often characterized by extended submission to acceptance and submission to publication times and lower number of citations as compared to other types of articles, in particular those having an empirical part or those focused on applications (Hamermesh, 2018). Hamermesh (2018, p. 119) states that "searching for citations using the Google Scholar database has (...) the advantage over the Web of Science database (WoS) of allowing citations to junior scholars to be available earlier in their professional careers, which is especially important given the long publication lags in economics that lead to lags in WoS citations."

- 14 talks given at the international scientific conferences abroad,
- 7 talks given at the international conferences held in Poland,
- 4 talks at national conferences held in Poland,
- 2 talks at the external research seminars abroad at the invitation of the organizers (the Economic Seminar Series, University of East Anglia, Norwich, June 8, 2009; the Management and Behavioral Research lunch series, Pompeu Fabra University, Barcelona, February 28, 2018),
- 2 talks at the external research seminars in Poland at the invitation of the organizers (Financial mathematics seminar at the Institute of Mathematics of the University of Silesia, 2015; XXIX conference of the Faculty of Economic Sciences at the University of Warsaw, plenary lecture, Chęciny, 2018),
- 1 poster presentation at the conference abroad.

I presented my results, among others at the following conferences:

a) *LabSi conference* in Salerno,
b) *European Doctoral Program in Quantitative Economics Jamboree meeting* (Paris, London),
c) *European University Institute Alumni Conference* in Florence,
d) *World Congress of the Game Theory Society*,
e) *European Workshop on General Equilibrium Theory*,
f) *Hurwicz Workshop on Mechanism Design Theory* in Warsaw (5 times),
g) *Foundations of Utility and Risk* (Tinbergen Institute, Warwick U, U of York),
h) *Ward Edwards Bayesian Research Conference* in Fullerton (4 times),
i) *Pan Pacific Game Theory Conference* at Waseda U in Tokyo,
j) *Time, Uncertainties, Strategies* in Paris,
k) *Modeling of Preferences and Risk* in Ustroń.

Articles written in co-authorship were presented at many international scientific conferences by my co-authors. For example, **Baucells, Kontek, and Lewandowski (2018)** was presented at the plenary lecture of *The 7th Xiamen University International Workshop on Experimental Economics* by Manel Baucells.

I was a member of the Organizing and Scientific Committee of *the VII Hurwicz Workshop on Mechanism Design Theory* (Warsaw, 2018), which was a great success.

In 2015-2016 I organized *Decisions - theory and experiments* – the research working group, and then from 2016 onwards I coorganize *Theory and decisions* as one of two parts of *Warsaw Economic Seminars* – the joint research seminar of SGH and the Department of Economics of the University of Warsaw, where scientists from Poland and abroad present their research in the areas of: economic theory, decision theory, game theory and experimental economics. In 2018, as part of this seminar, I organized a public lecture by prof. Eyal Winter from Hebrew University in Jerusalem entitled "Feeling smart. Why our emotions are more than we think".

I reviewed articles submitted for publication in a number of highly recognized scientific journals, including:

- *Management Science* (IF: 3,544; MNiSW: 40) - 1 article;

- *Games and Economic Behavior* (IF: 0.878; MNiSW: 30) - 1 article,
- *Theory and decision* (IF: 0.522; MNiSW: 25) - 1 article,
- *Economics Letters* (IF: 0.581; MNiSW: 20) - 1 article,
- *The Geneva Risk and Insurance Review* (IF: 0.313; MNiSW: 20) - 1 article,
- *Journal of Behavioral and Experimental Economics* (IF: 0.966; MNiSW: 15) - 1 article,
- *The BE Journal of Theoretical Economics* (IF: 0.220; MNiSW: 20) - 2 articles.

I currently participate in two research projects/grants:

- From 2018, I am the leader of the research project entitled "**Modeling of preferences in the field of risk and uncertainty, including dependence on range, rank and reference point".** This project is part of the statutory research at the Collegium of Economic Analysis of SGH. As part of this research, I analyze deviations from game theoretic prediction in simple games using *GRows* – a mobile application especially designed for this purpose.
- From 2016, I am the investigator in the research project entitled "**Methods of analysis of decisions in multi-criteria problems and estimation of willingness to pay/accept using fuzzy modeling**". This is the OPUS 10 grant awarded by the National Science Center and led by Michał Jakubczyk.

As part of work for public administration, I participated in several projects commissioned by public administration authorities:

- 2016-2018: I was the leader of the project entitled "**Identification of regional diversification sources in Poland using the decomposition of GDP per capita growth and GVA per capita differences**". This project is part of the program Statistics for Cohesion Policy financed from the Operational Program Technical Assistance.
- 2013-2015: I was the leader of the project entitled "**Methods of decomposition of GDP per capita growth and GVA per capita differences as applied to the analysis of the structure of regional differences in Poland**". This project is part of the program Statistics for Cohesion Policy financed from the Operational Program Technical Assistance.
- 2011, I was the author of the expert opinion entitled "**Identification of determinants of economic development of Eastern Poland using the decomposition method of GDP**", commissioned by the Ministry of Regional Development.

Each year since 2012 I write a chapter on the methods of the decomposition of growth, differences, and inequality according to Theil's coefficient in the annual Report entitled *Macroeconomic situation in Poland in the context of the world economic processes* published by *Statistics Poland*. I have repeatedly presented the decomposition methodology at public administration and scientific conferences, including e.g. "Social challenges of statistical education" in 2016 in Gdynia.

I conduct trainings for public administration employees, such as: *Pitfalls of statical thinking. How to draw correct conclusions using probabilities and statistics*, *Data classification using data mining methods according to the CRISP process*, *Decomposition methods*.

In 2017 and 2018, I served as member of the jury of the Central Statistical Olympiad final stage. I supervised *Brain club* – the scientific student circle at SGH. I also served as the elective of the Collegium of Economic Analysis of SGH at the Rector's election. I received prizes and congratulatory letters from the Rector of SGH for scientific and organizational activities.

At SGH, I teach and have taught a number of courses: advanced microeconomics (master level), game theory (PhD, master), decision theory (PhD, master, bachelor) and operations research (bachelor). Most of my courses are original lectures run in English. I also taught lectures in math for economists (PhD level) at the European University Institute in Florence (2008-2009) and operations management (bachelor and master level) at the Leon Koźmiński University in Warsaw (2017-2018).

I have supervised 19 master's theses and 123 bachelor's theses. I am the author of three original lectures and two case studies.

In addition to strictly scientific activities, I also popularize science. In 2016, I performed as an expert in the radio program entitled *From another planet – Game theory* in „Radio dla Ciebie". In 2015, I published an article entitled *Are Pacifists Right?* in „Egzorcysta" monthly.

My scientific achievements, along with didactic and organizational achievements, are described in more detail in Appendix 4 to the application.

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