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THE INFLUENCE OF A VOTING METHOD ON INDIVIDUAL AND COLLECTIVE DECISION MAKING IN THE CONTEXT OF PARTICIPATORY BUDGETING

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Summary of a doctoral dissertation
written under the supervision of
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Warsaw, June 2022

Introduction

The main goal of this thesis is to understand the influence a voting method can have on the behavior of individuals and the resulting social outcomes in participatory budgeting elections.

Participatory budgeting (PB) is a process allowing citizen engagement in public funds' allocation to better represent the needs and preferences of the communities. Since its introduction in Brazil in the 1980s, the PB idea has spread across the world, becoming a significant area of innovation and development for local democracy (Cabannes, 2004). Although a PB process can be executed in many ways, its collective decisions are often made through voting. That is, the residents cast their votes, which are then counted to select a winning set of projects subject to a budget constraint.

The choice of a ballot type determines how voters can express their preferences, and the scoring rule indicates how their votes are then translated into the election results. Both of these factors can affect the individual voting strategies and social outcomes and are examined in this thesis. Our study focuses on two methods used in the context of participatory budgeting. The first one is knapsack voting, where voters select as many projects as they wish as long as their total cost does not exceed the dedicated budget. The second one is k-approval voting, where they select up to a certain fixed number of projects without any limitations imposed on their costs.

Our research is formally embedded in social choice, which provides a theoretical framework for translating individual preferences into a group decision. Aggregate assessment and the question of how to determine a collective outcome with a variety of people's possibly diverse needs and interests are central to social choice theory (Sen, 1970). Consequently, designing a voting method, that is a practical representation of a social aggregation function, is a high-complexity task and often incorporates making difficult tradeoffs. In fact, Arrow (1951) has shown in his Impossibility Theorem that certain plausible fairness criteria cannot be simultaneously satisfied by any procedure. Further, Gibbard (1973) and Satterthwaite (1975) proved that there was no voting rule immune to strategic manipulation. That is, no matter how simple or complicated a rule we can think of, it will never pass all the desired conditions and there will always be an incentive to vote insincerely. Social choice, with its formal tools and techniques, helps to avoid or solve many of the theoretical issues and inconsistencies. However, it is through empirical observations and experiments that we learn about the practical performance of voting methods and additional challenges to their implementation.

The organization of the thesis is as follows. In Chapter 1 we formally introduce the relationship between individual values and social welfare (Mas-Colell et al., 1995). We also describe several popular ballot types and aggregation rules to show that different voting rules can lead to different outcomes (Nurmi, 2010). Chapter 2 contains a review of laboratory and field experiments on individual voting strategies under different voting systems and the effects they have on the election results. In Chapter 3 we define the participatory budgeting problem with different approaches to preference elicitation, utilities' modeling and welfare maximization (Aziz & Shah, 2021). We also report on related research regarding axiomatic properties, computational efficiency, and empirical performance of different voting rules applicable in the PB context. Chapter 4 focuses on the comparative analysis of empirical data collected in the actual Warsaw Participatory Budget in the context of the change from knapsack to k-approval voting implemented between its 4th and 5th editions. Chapter 5 reports on the design and results of a complementary online voting experiment conducted to verify a number of research hypotheses relating to individual voting strategies under knapsack versus k-approval and the structure of the winning sets produced by alternative ballot types and aggregation rules. The research in these last two chapters was supported by the Doctoral Scholarship of the Capital City of Warsaw and its preliminary results are available in the project's report (Szczupska, 2021). The thesis ends with summary of the results and conclusions.

Related literature review

Our review of individual voting behavior and collective outcomes under different voting situations is based on the following two types of experiments. The first one, a classic laboratory approach, allows us to closely observe voters' monetary-induced behavior in highly controlled conditions. The second one, field experimentation, enables us to get in touch with a larger electorate in an environment as close as possible to the real elections.

Several voting rules were tested in the reported experiments. The most common in real-world elections are uninominal rules, where voters are asked to select a single candidate. Examples of those are plurality, where an election is won by the candidate with the most votes, and plurality runoff, where the second round enables a decision between the top two candidates if none of them obtains an absolute majority of the votes in the first round. In the less often encountered multi-nominal rules, voters can select multiple candidates. Their prominent

examples are approval voting (AV) and Borda count. Under the former, proposed by Brams & Fishburn (1978), voters select any number of alternatives they approve of, and the one with the largest number of received votes wins. Under the latter, introduced by Borda (1781), every voter gives a points to the least preferred alternative, $a+b$ to the next one, $a+2b$ to the third worst, etc., and the one with the highest total score wins. Other experimentally studied methods included evaluative voting (EV) and single transferable vote (STV). Under EV, voters are asked to provide a score from a given set for each candidate and the one with the highest total score wins. Under STV, voters are asked to rank the candidates. If no candidate receives more than half of first-rank votes, a specified elimination procedure transfers votes from those with the smallest number of first ranks, or the highest number of last ranks, until a (winning) candidate obtains the required majority support. Some methods which use verbal scale grades rather than numerical ratings or rankings were also studied. In dis&approval voting (Alcantud & Laruelle, 2014) the voter preferences are reported as "In favor", "Against", or "Neutral", the candidates then receive one, minus one, or zero points respectively, and the one with the highest total score wins. Finally, in the majority judgment (Balinski & Laraki, 2007) voters give one of the five grades from "Excellent" to "to Reject" to each candidate, and the one with the highest median grade is elected.

In the laboratory settings, the results demonstrated that voters behave differently depending on the amount and type of information they had and the level of uncertainty they faced (Forsythe et al., 2006; Kube & Puppe, 2009), as well as how much and what kinds of cognitive effort was needed (Malawski et al., 2010; Przybyszewski et al., 2011). Voters acted strategically under simpler, more intuitive rules, such as approval and plurality. They would rather switch to sincere voting with sophisticated reasoning and computations under STV. Relying on simple heuristics, that is using efficient shortcuts, gained relevance in situations of moderate complexity, like plurality with runoff (Van der Straeten et al., 2010).

The comparison of the outcomes demonstrated that different voting methods led to different winners. Overall, the multi-mark ballots were more effective at electing a Condorcet winner (Blais et al., 2007; Igersheim et al., 2016) and not electing a Condorcet loser (Forsythe et al., 1993). These are candidates who would have, respectively, won and lost a pairwise contest with any other candidate.

The field experimentation was pioneered by Balinski et al. (2003) and Laslier & Van der Straeten (2004) who tested large-scale approval voting during the first round of the 2002

French presidential elections. Since then, the approach has been adopted in many other experiments. In France, the tradition continued with the presidential elections of 2007 through 2017. Apart from the approval voting, the following methods were studied: majority judgement (Balinski & Laraki, 2011), single transferable vote (Farvaque et al., 2009), evaluative voting (Baujard et al., 2011, 2014), and dis&approval voting (Laruelle, 2018). Similar experiments have been also realized in other countries, for example, during official state and federal elections in Germany (Alós-Ferrer & Granic, 2010, 2012), or via polls preceding presidential elections in Poland (Przybyszewski & Sosnowska, 2006, 2016) and the United States (Igersheim et al., 2021).

General learning from these experiments was that testing new voting methods on a larger scale and with a more representative sample than in the laboratory conditions was possible. The participants showed had a good understanding of the alternative rules and made extensive use of the additional expressive possibilities offered by the experimental multi-mark ballots. They engaged in little or no strategic behavior, as proven by their heavy use of intermediate ratings and a reluctance to give negative scores.

Comparing the outcomes of the experimental elections led to the conclusion that the same electorate can produce different outcomes under different methods. Both the ballot type and the aggregation rule were shown to be important factors. Overall, compared to the official plurality rule, the alternative multi-nominal methods favored inclusive candidates while disadvantaging the more extreme ones. They also revealed a true level of support for minor candidates in hypothetical large-scale elections, providing a better overall reflection of the political landscape as perceived by the voters.

With a well-established theoretical and empirical evidence that a voting method influences the behavior of individuals and the resulting social outcomes in single-winner elections, we now move on to the case of participatory budgeting, a multi-winner setting with an additional budget constraint. Our focus is on the related results based on simulated, experimental, and real-life PB datasets.

First, the research showed that simpler input formats, such as k-approval, knapsack, or straightforward ranking were less cognitively demanding, as indicated by the measured response time (Benade et al., 2018). Value-for-money ranking seemed more burdensome but was proven to work well if executed through pairwise comparisons. More complicated formats, such as cumulative votes (where each voter is given one coin and asked to specify how it should

be split among the projects), showed promising theoretical performance, but it is yet to be determined whether they require an acceptable voting effort (Skowron et al., 2020).

Second, the empirical data showed a significant impact of having project cost consideration built into the voting method. Namely, the knapsack ballot with its embedded budget limit led to a more economical selection of the projects compared to k-approval (Goel et al., 2019).

Once the votes are cast in a given format, the information they carry is fed into the aggregation function, and a decision has to be made on how the residents derive satisfaction from the selected projects. For example, whether it is just about having any of their supported projects funded, depends on the number of supported and funded projects, or their costs also play a role. Again, the assumptions we make here were shown to impact the outcomes (Talmon & Faliszewski, 2019).

Finally, solving the participatory budgeting problem is about selecting a set of winning projects that maximizes social welfare. However, several approaches to welfare optimization exist and different aggregation algorithms can be applied in practice. It should not come as a surprise at this point that the research implied both will affect the outcomes of the elections. For example, it was shown that the winning sets produced under utilitarian and Nash welfare were judged by the voters as the most appropriate (Rosenfeld & Talmon, 2021). From the implementation point, greedy iterative algorithms are most often used in practice, due to their significant computational advantages. Their cost-proportional version, as empirically verified (Laruelle, 2020), leads to funding a larger number of cheaper projects.

These reported empirical results are fairly recent and still scarce within the social choice participatory budgeting literature, which is predominantly focused on axiomatic and algorithmic approaches. Our study, through its unique combination of the findings from the real Warsaw PB elections and the online PB voting experiment, can offer valuable contributions to the field.

Research methodology and hypotheses

Leveraging the occurrence of a natural experiment to empirically evaluate the change of a voting system in actual large-scale PB elections is a novel proposition, to our best knowledge, not found in the relevant existing literature. Such an approach provided a distinctive

opportunity to gain access to the entire population of interest in a real voting situation, with the true motivations and preferences of the electorate. Four consecutive editions of the official Warsaw PB were studied, with two conducted under knapsack and the other two under k-approval voting. The real-life empirical settings with multiple variables outside of our control posed certain limitations to the comparative analysis of the studied editions.

To begin with, Warsaw PB is already conducted separately in each of the city's districts. Thus, there are numerous respective elections with their own project lists, budgets, votes, and outcomes to consider and reconcile. Furthermore, in parallel with the change of the voting system between the 4th and 5th edition, other modifications were implemented. Most notably, the existing division into smaller sub-districts was discontinued, and a new city level was introduced instead, which led to an increased number of projects on the ballots, as well as their bigger scale and higher cost. The above posed challenges to an effective estimation of naturally overlapping effects in isolation. Proper assumptions and techniques had to be applied when analyzing the data and interpreting the results. For example, to study a potential bias among voters toward more expensive projects under k-approval, where cost consideration is not built into the rule, both the projects' cost and vote shares had to be normalized to allow a fair comparison of different elections within and across the studied editions.

The evaluation of Warsaw residents' voting strategies and the election outcomes is complemented by the results of a voting experiment conducted via the Stanford Participatory Budgeting Platform in cooperation with the Capital City of Warsaw Municipal Office. The data was collected through an online survey, where participants (180 Warsaw residents recruited through the Municipal Office website and additional social media channels) were asked to vote under both the knapsack and k-approval method in a randomly assigned order. The fictional PB elections were designed to mimic the real Warsaw PB. A common list of projects consisted of official city-level projects from previous editions with their actual costs and descriptions, and the same budget limit was set in both scenarios. A voluntary voter experience poll was included at the end, asking participants to rate the tested methods in terms of their perceived simplicity, the required effort to cast a vote and a level of cost consideration involved, as well as their overall preference towards the methods being used in the official Warsaw PB.

The related literature review and empirical observations helped form the following six main hypotheses to be verified by the experimental research.

H1. The number of projects selected by voters under knapsack voting is different than the number of projects supported under k-approval.

H2. Voters are more economical in their projects' selection under knapsack voting as compared to k-approval.

H3. The knapsack ballot produces a different winning set of projects than k-approval.

H4. A modification of the aggregation rule changes the collective outcomes.

H5. There is no difference in voters' subjective perceptions of knapsack versus k-approval voting.

H6. A higher ballot position leads to greater support for a given project.

Summary of the study results

The first hypothesis, that the number of projects selected by voters is different under knapsack voting than under k-approval, was confirmed by the experimental data. In the case of our online voting experiment, significantly more projects were supported under knapsack compared to 10-approval. However, the empirical observations from Warsaw PB indicated that more projects were selected in both k-approval editions compared to the knapsack ones. The difference in the direction can be explained by external factors, such as the amount of dedicated budget compared to the individual projects' costs for knapsack ballots and the choice of k for the k-truncated approval ballots. We have also noted a clear trend among voters in both the real and experimental Warsaw PB to select the maximum allowed k projects in the case of k-approval voting. These results suggest that for many residents, the projects selected with an unrestricted number of approvals might be different than when the number is fixed at the maximum of, for example, 10 or 15. Furthermore, despite a non-negligible amount of bullet voting in the real elections, supporting one project only was barely present under the experimental conditions. This hints that such a strategy was used rather as a (potentially coordinated) effort to increase the most preferred project's chances to win, and not as a cognitive shortcut in response to a demanding voting procedure.

The second hypothesis stating that voters are more economical in their projects' selection under knapsack compared to k-approval voting was supported by the results of the experiment and

the empirical evidence from the real Warsaw PB. Whether we consider the cost of all supported projects or the average cost of a single supported project, these under 10-approval experimental scenario were significantly higher compared to the knapsack scenario. Correspondingly, under k-approval in both the 6th and 7th editions, there was a significant positive association between normalized costs of projects and their vote share index, indicating a surplus of votes cast for relatively more expensive projects. Under knapsack in the 4th and 5th editions, the popularity of projects was quite equally distributed by their relative costs. An observed surplus of votes cast for the cheapest projects was related to a common strategy among the residents of additionally selecting cheap projects to use up the full dedicated budget.

The third hypothesis, about different winning sets being produced under knapsack compared to k-approval, was confirmed by the experiment. The results revealed significant effects of the tested ballot type on the selection of projects to be funded. These outcomes were also qualitatively different. Under knapsack, twice as many but half as costly projects were voted to be funded. Such a large magnitude of the change in average funded project costs (and consequently their total number) can still lead to meaningful practical impacts, despite the differences not being found to be statistically significant. Comparing the outcomes of four studied editions of Warsaw PB, we have also noted more and cheaper projects winning under knapsack versus k-approval ballots. In particular, in the 4th and 5th knapsack editions, the cheapest projects were overrepresented among funded compared to not funded projects, while the more expensive ones were underrepresented. In the 6th and 7th k-approval editions the cost-related structure of projects that won closely mirrored the one of those that lost.

Both types of ballots provided comparable levels of voter representation defined as the fraction of those for whom at least one of the supported projects got to be funded. However, voting under the experimental knapsack scenario led to significantly higher average voter satisfaction scores compared to the experimental 10-approval conditions. This applied to both the item-based satisfaction defined as a ratio of the number of projects supported and funded to the number of all supported projects, and cost-based satisfaction derived from the relative cost of the supported and funded projects to the total cost of all supported projects. The identified satisfaction scores' patterns across the two studied methods were directionally in line with the ones observed in the real Warsaw PB elections.

The fourth hypothesis that stated a modification of the aggregation rule leads to a change in the collective outcome was validated by the results of what-if scenarios analysis performed on the

experimental data. First, we recalculated the two original results by applying the satisfaction approval voting (SAV) approach to the existing ballots, that is assuming every voter essentially splits their vote evenly between all their supported projects (Brams & Kilgour, 2014). With some minor changes observed in the winning sets, there was no significant impact of applying SAV calculations to either knapsack or k-approval ballots under the default greedy algorithm. Next, these four obtained datasets (two original and two based on SAV) were aggregated applying the cost-proportional greedy algorithm. The same winning set was produced as a result, but it was significantly different for all four pairs of the tested what-if scenarios. Under the cost-proportional algorithm, the number of funded projects was relatively the highest and their cost the lowest. An average funded project cost was close to one-fourth of those selected under the original 10-approval scenario that were overall the most expensive. While these differences were not found to be statistically significant, they still revealed a non-negligible practical impact.

The fifth hypothesis stating there is no difference in voters' subjective perceptions of knapsack and k-approval voting did not find support in our survey data. The voters did not see any significant difference in the comprehension of the tested methods, finding both very simple. However, casting a knapsack vote was perceived as a significantly harder procedure than casting a 10-approval vote. These subjective effort scores of the former were particularly high when the knapsack votes were cast as second in the experiential setup, that is following the 10-approval vote. The extra effort required under knapsack was linked to the reported more economical consideration of the projects and it was not necessarily perceived as a disadvantage. In fact, voters expressed their clear preference for knapsack to be used in the real Warsaw PB elections. Further, in their qualitative feedback, they also articulated appreciation of the knapsack ballot and its budget limit for a built-in nudge to make more thoughtful decisions.

The sixth hypothesis about a higher ballot position leading to greater support for a given project was not confirmed in the experimental conditions. There was no significant association between the position on the ballot and the popularity of the projects for neither knapsack nor k-approval scenario under the common list of 50 projects. These conclusions differed from what we observed in the 6th and 7th (k-approval) editions of real Warsaw PB, where significant ballot position advantage was found in the aggregated analysis of district and city project lists. A further investigation revealed that these effects were predominantly driven by the longest ballots with 100 projects or more and were also dependent on the ballot length itself.

Concluding remarks

In this thesis, we have shown that group decision making is a high-complexity task if one wants to fairly represent the needs and interests of all the individuals. Not only there are multiple ways of eliciting personal preferences to choose from, but also a variety of aggregation procedures that can be applied once the votes are cast. We recognized that even for single-winner elections there is no voting method satisfying all desired prespecified properties. In the case of participatory budgeting, a multi-winner setting with an additional budget constraint, the challenge of designing and implementing the best possible voting method only increases.

The results of our research, supported by the review of existing literature, showed that difficult trade-offs are often required between theoretical properties and practical applications of alternative solutions. For example, collecting comprehensive information on voters' preferences might be desired from the modeling point of view. However, in reality, the complexity of a ballot has to be properly balanced with an acceptable cognitive burden imposed on a voter. This is particularly relevant for PB settings where we commonly encounter long ballots with a hundred projects or more.

Another important decision relates to the consideration of project costs being ingrained either in the ballot type, as is the case of the knapsack vote and its budget limit, or built into the aggregation rule, as is the case of the cost-proportional algorithm. A bias toward cheaper projects when cost consideration is required by the method is consistently found in the related literature, our empirical observations, and experimental results. Such methods, whether through voter behavior, a scoring rule, or both, lead to a selection of projects with relatively lower costs. Given a fixed budget, more of them can be funded as a result, offering a chance for higher voter representation and satisfaction.

Since different participatory budgeting voting methods can lead to different winning sets of projects representing the interests of the same group of residents, the choice of a ballot type and aggregation rule should receive serious consideration in the PB process. Our findings could serve as practical guidelines for the design and implementation of desirable PB solutions, as well as their post-election evaluation. We would like to emphasize that the thesis does not provide normative recommendations but presents certain properties of voting methods that might be viewed as beneficial in some situations and disadvantageous in others.

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