Warsaw School of Economics
Collegium of Economic Analysis
Institute of Econometrics

Econometric modelling of sovereign risk

Konrad Kostrzewa

Summary of doctoral thesis

written under supervision of

dr hab. Dobromił Serwa

Warsaw, 2023

The dissertation thesis reflects on the econometric modelling of risk that is related to pricing and managing portfolios of sovereign bonds and Credit Default Swaps (CDSs). The modelling of sovereign CDS' risk, as well as their relationships with other assets, serve various market participants (e.g., investment funds, banks, and financial market supervisors). Sophisticated econometric techniques can be an advantageous resource, especially for market makers. These methods allow them to calculate the risks associated with their portfolios with a higher degree of precision; this can lead to both profits from fluctuations in market prices and increased liquidity for other participants.

The key objective of this dissertation thesis is to provide comprehensive research on whether proposed advanced econometric methods allow for more accurate risk estimations than do historical methods. This thesis empirically validates three main hypotheses, namely:

1. Including a volatile Loss Given Default (LGD) over time in the CDS pricing model significantly improves estimations of Probability of Default (PD) and the fit to data;
2. Taking into account a non-linear relationship between SCDS and an underlying bond using copula-based functions that significantly improve risk estimates of the portfolio consisting of both a bond and CDS;
3. Neural network forecasting models significantly improve the forecasts of sovereign CDS volatility;

The thesis consists of four chapters. The first chapter describes CDS contracts' definition, purpose, and the global market for them. This chapter focuses on market participants, their structure, and the evolution of the market over time—along with the situation during the European sovereign debt crisis. The second chapter sets out a proposal for a CDS-pricing model with time-varying PD and LGD while simultaneously using the term structure of contracts. The reduced-form model that is used to establish the price of Polish sovereign CDS contracts has been evaluated. The outcome indicates that the model has accomplished an improved alignment between the estimated values and the factual data when compared to the more concise specifications. The robustness of the results has been confirmed with two other European countries: France and Spain. The third chapter presents a copula-based method of estimating risk of a portfolio that consists of a sovereign bond and a CDS on that bond. The chapter analyses the relationship between a sovereign CDS contract and the underlying bond, and validates the hypothesis on whether taking into account a non-linear relationship between them using copula-based functions significantly improves risk estimates. The chapter concludes that the proposed method is at least as good as the correlation-based method and in some cases leads to statistically superior results. The fourth chapter introduces a hybrid model combining generalised autoregressive conditional heteroscedasticity (GARCH)-type and neural network approaches and validates the hypothesis of the superiority of neural networks over classical models for volatility forecasting. The results of a rigorous evaluation of the predictions produced by GARCH-type models and a hybrid model indicate that, for extended forecasting periods (five and 15 days), the hybrid model surpasses both the naive forecasts and the forecasts of GARCH-type models. However, for short forecasting periods, the naive forecast remains the most effective. These findings have been verified across various measurements of volatility, thereby demonstrating their robustness. The final chapter concludes the results of the thesis.

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Topic, aim and scope of the thesis

The dissertation thesis reflects on econometric modelling of risk related to pricing and managing portfolios of sovereign bonds and Credit Default Swaps (CDSs). The key objective of this dissertation thesis is to provide comprehensive research on whether proposed advanced econometric methods allow for more accurate risk estimations than do historical methods. This thesis empirically validates three main hypotheses, namely:

1. Including a volatile Loss Given Default (LGD) over time in the CDS pricing model significantly improves estimations of Probability of Default (PD) and the fit to data;
2. Taking into account a non-linear relationship between sovereign CDS and an underlying bond using copula-based functions significantly improve risk estimates of the portfolio consisting of both a bond and CDS;
3. Neural network forecasting models significantly improve the forecasts of sovereign CDS volatility;

Each hypothesis is thoroughly analysed in a separate chapter, where an appropriate methodology has been selected based on the literature review. Moreover, each hypothesis is validated using a different dataset. The set of countries and the time period analysed in each chapter is selected in a way that allows one to draw meaningful conclusions while, at the same time, maintaining a common factor for all of the main hypotheses. Analysed hypotheses relate to the problems that apply to all market participants that are active on the sovereign CDS market. When hedging their position with a CDS contract the issues of correct pricing, the future risk of the instrument, as well as the risk of the whole portfolio are factors of major interest and must be thoroughly analysed.

This study contributes to the existing literature in several ways. Firstly, not only does it estimate the time-varying PD and LGD simultaneously using the term structure of sovereign credit default swap contracts, but it also compares the proposed model with other pricing models known in the literature. It reveals severe deficiencies in the current asset pricing approaches to sovereign CDS where assumed recovery values are far from historical observations and the market expectations. Secondly, it describes a methodology that allows for assessing the risk of a portfolio that consists of a sovereign bond and a CDS on that instrument, as well as presents conclusions on the effectiveness of such a methodology. The study compares a wide range of copula functions, and validates whether more advanced methods of portfolio risk assessment allow for the procurement of more accurate risk estimates. Finally, it applies sophisticated, advanced methods (e.g., neural networks and nonlinear state space models) that capture non-linearities to the CDS market and compares them to more common and less computationally demanding equivalents.

The modelling of sovereign CDS' risk, as well as their relationships with other assets, serve various market participants (e.g., investment funds, banks, and financial market supervisors). Sophisticated econometric techniques can be an advantageous resource, especially for market makers. These methods allow them to calculate the risks associated with their portfolios with a higher degree of precision; this can lead to both profits from fluctuations in market prices and increased liquidity for other participants. It is especially important for those market participants (reporting dealers and central counterparties) to manage their risk. This is because they are both global and systemically important financial institutions who dominate the buy and sell sides of that market. If these institutions inaccurately assess the risks during turbulent times, it could result in systemic events.

The topic is also crucial for all financial institutions investing in sovereign bonds: banks, investment funds, hedge funds, insurance companies etc. These institutions rely heavily on risk estimates for those instruments and their derivatives. Sovereign bonds for many financial institutions in Poland remain a dominant element of their assets. They account for 11% of all assets of the Polish banks[1], 20% for investment funds, 69% of life-insurance companies' assets and 52,2% of non-life insurance companies' assets[2]. The significant proportion of those assets in the sector is additionally important as Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms introduced an obligation on daily reports on risk (Regulation (EU) No 575/2013).

Conclusions of this thesis should provide insights on possible application of researched econometric methods to analyse the sovereign debt market. Moreover, they can be used by market participants to improve their pricing and risk assessment models. The policymakers, however, can interpret the results to gain insight on the market risk assessment, use the proposed methods as early warning tools, or even take them into consideration when formulating the recommendations to institutions.

The thesis is composed of four chapters which are described below.

<u>Chapter 1: Credit Default Swap characteristics and market description</u>

The chapter describes CDS contracts' definition, purpose, and the global market for CDS instruments. It focuses on market participants, their structure, and the evolution of the market over time—along with the situation during the European sovereign debt crisis.

Credit Default Swaps (CDS) are financial instruments that transfer credit risk from one party (buyer) to another (seller). In a typical bond contract, the issuer (the reference entity) commits itself to paying its investors a regular fee, as well as to redeem the principal at the maturity of the contract in exchange for the principal amount. As a result, the investor is exposed to the credit risk because it bears the risk of capital losses in case of the issuer's default.

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[1] Own calculations based on data from National Bank of Poland.
[2] Report on Financial System Development, NBP 2020

A CDS is a derivative instrument that acts as a form of insurance against the issuer's bankruptcy. The buyer pays periodic payments (premiums) to the seller but will receive compensation for the losses caused by the reference enitity's default or restructuring. On the other hand, the seller receives premiums but must cover the losses in case of a default. The CDS mechanism is presented in Figure 1.

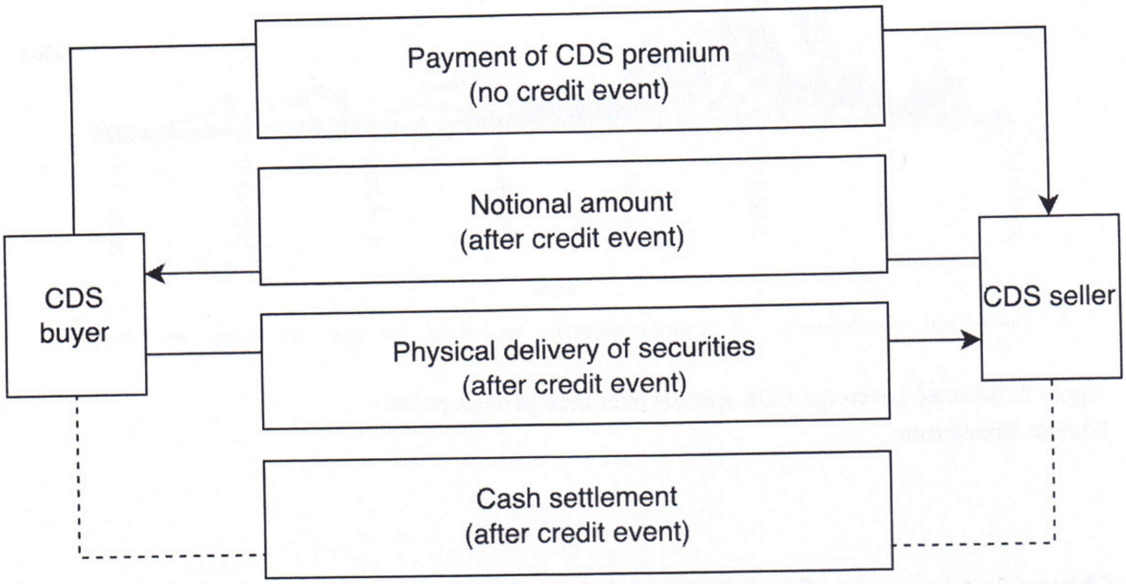

Figure 1: CDS payment mechanism.
Source: own work based on Criado et. al. (2010)

Credit Default Swaps as financial instruments were created in the second half of the 1990s and were firstly traded in London. Later they were also traded in major economies, particularly in the United States. From the early 2000s the CDS market started growing rapidly as investors saw them as both hedging and speculation tools. The market grew exponentially and its growth lasted until the collapse of Lehman Brothers in 2008 (Tamakoshi & Hamori, 2018). Due to the fact that Credit Default Swaps played a major role in the buildup of the financial crisis, the popularity of the instruments fell drastically.

The Sovereign Credit Default Swap (SCDS) market is relatively new and small (USD 1.1 trillion gross national as of end March 2021) in comparison to USD 8.8 trillion gross national value of the total CDS market globally. The SCDS market has been characterised by the rapid growth from 2005, reaching its peak between 2012 and 2013 during the European sovereign debt crisis caused by the financial situation of Greece in 2009. During that period SCDS spreads for bonds issued by European countries have reached their historical maximas. Values of spread for selected countries are presented in Figure 2.

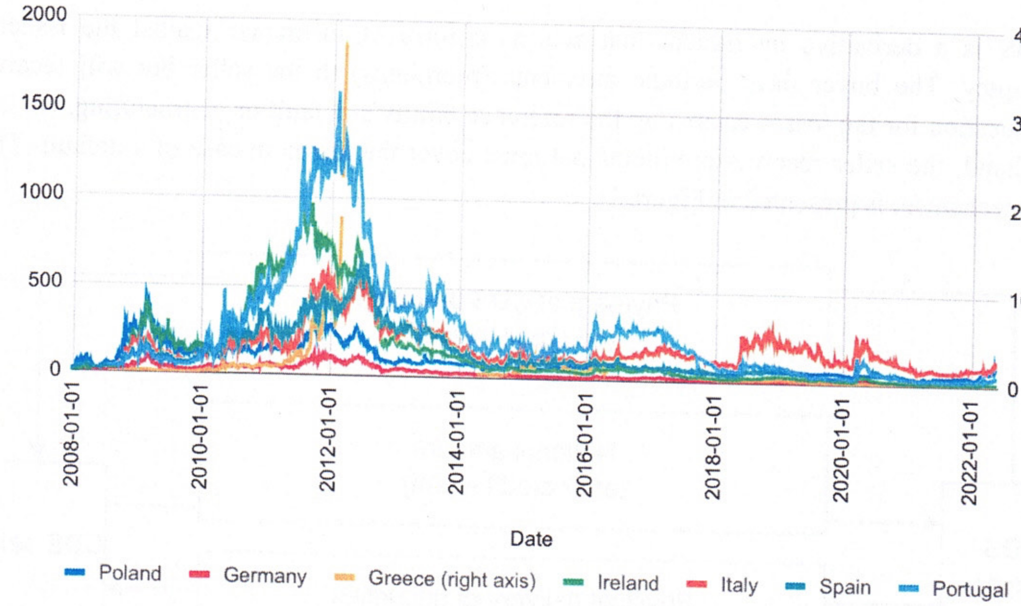

Figure 2: Selected sovereign CDS spreads over time in basis points.
Source: Datastream

Chapter 2: Assessment of probability of default (PD) and loss given default (LGD) from the quotes of sovereign CDS contracts

The chapter sets out a proposal for a CDS-pricing model with time-varying PD and LGD while simultaneously using the term structure of contracts.

The main risk factor of debt securities is the issuer's inability to repay the debt. The higher the likelihood of the institution's default, the cheaper should be the security of the given nominal value. Similarly, the price of a debt security depends on the amount the investor can recover in case of the credit event. The higher the percentage possible to recover, the safer is the instrument and the price should be higher.

Having in mind the size of debt securities market and the influence of described factors on their price and risk management, the topic was researched in many publications like Gapen et al. (2008), Plank (2010), Trebesch et al. (2012) or Konopczak (2014). In case of sovereign debt securities credit events happen very rarely and as a result the estimation is impossible using historical data. In practice models assuming a constant LGD and time-varying PD based on CDS spreads or CCA (Contingent Claims Analysis) are used. The assumed level of LGD can highly influence the final estimate of the PD and as a result the price of the security based on this value. Moreover, historical data show that LGD can vary between various cases (Trebesch et al. 2012), which proves the need to estimate that value. The chapter describes a model based on work of Pan and Singleton (2008) and Doshi (2011) which takes advantage of the state space modelling techniques and the Kalman filter to estimate time-varying market participants' expectation towards both PD and LD.

Moreover, using the model based on CDS spreads allows to overcome difficulties resulting from low liquidity of the bond market. According to Longstaff et al. (2011) the CDS market is usually more liquid than the corresponding bond market. As a consequence, estimates based on the CDS spreads should be more reliable.

The chapter presents results for Poland, a relatively large economy in Central and Eastern Europe. Due to the fact that it did not suffer from the macroeconomic and financial imbalances that characterised many emerging and developed economies in the years before the global financial crisis caused by the fall of Lehman Brothers, but still was affected by the global financial turbulence, Poland may serve as a natural laboratory for studying the transmission of risk. The aim of this chapter is to show that the model that assumes a time-varying LGD can be superior in fit to the empirical data.

The reduced-form model presented in the chapter has been evaluated using month-end observations collected from January 2004 to December 2021 for 10 different maturities, ranging from one to 10 years. The robustness of the results has been confirmed with two other European countries: France and Spain.

<u>Chapter 3: Sovereign bond-CDS portfolio risk modelling with copulas</u>

The chapter presents a copula-based method of estimating risk of a portfolio that consists of a sovereign bond and a CDS on that bond. It analyses the relationship between a sovereign CDS contract and the underlying bond, as well as validates the hypothesis on whether taking into account a non-linear relationship between them using copula-based functions significantly improves risk estimates.

Credit Default Swap (CDS) is an instrument allowing the owner of the bond to hedge the risk of the issuer's default. Bonds issued by most countries were considered very safe for a prolonged period of time. At the same time prices of CDSs on those bonds were low. As a result, when assessing the risk of a portfolio consisting of a sovereign bond and a CDS on that bond, the main focus was on the price of the bond, as the CDS was a neglectable part of the portfolio. As a consequence, in case of closing the position on both instruments, the profit or loss was almost entirely driven by the price of the bond.

During the period between 2009 and 2012 spreads of CDS on many European sovereign bonds have reached their historical highs: 1600 bp (Portugal), 1300 bp (Ireland) and 5000 bp (Greece). Under such scenarios the value of the CDS accounts for a large part of the portfolio and should not be neglected when modelling the risk of that portfolio. Profit/loss resulting from selling the bond and closing the position on the CDS when the CDS spread is high, still is mostly driven by the price of the bond, however the CDS significantly contributes.

Moreover, in 2012 in the EU the ban on holding a CDS when the entity does not hold a corresponding bond, was introduced. As a result every CDS on the european market is accompanied by the bond and as a result the modelling of the relationship became more relevant.

In the empirical section of the chapter marginal distributions from the wide spectrum of ARMA-GARCH models were fitted to the analysed time series. Applied GARCH-type models include the variations allowing the asymmetricity in the volatility response to positive and negative shocks such as EGARCH (Nelson, 1991) and GJR-GARCH (Glosten, Jagannathan i Runkle, 1993). Copulas are a powerful tool in financial modelling when it comes to take the lack of normality in log returns (Kharoubi-Rakotomalala, 2013). Therefore, for each pair of bond and CDS the optimal copula function was selected and its parameters were estimated. Based on estimated copulas and marginal distributions, 1-day VaR (Value-at-Risk) forecasts were calculated. The quality of the forecasts was compared using statistical tests: Kupiec (1995), Christoffersen (2004) and DQ (Engle i Manganelli, 2004). The chapter presents results for countries with highest CDS spreads during the European sovereign debt crisis: Greece, Ireland, Italy, Spain and Portugal, as well as for Poland for comparison.

Chapter 4: Forecasting the volatility of sovereign CDS: A hybrid model integrating LSTM with GARCH-type models

The chapter introduces a hybrid model combining generalised autoregressive conditional heteroscedasticity (GARCH)-type and neural network approaches and validates the hypothesis of the superiority of neural networks over classical models for volatility forecasting.

Modelling and forecasting of future volatility is crucial in risk assessment and portfolio management. Moreover, volatilities and their forecasts are often used in derivative pricing. It is a common market practice to apply one of the following methods to obtain a future volatility estimates: 1) extracting implied volatility from derivatives, 2) forecasting volatility using historical data. For less liquid and less popular instruments only the second method might be of use.

To model the discussed volatility a wide range of models have been developed since Robert F. Engle proposed an ARCH (Autoregressive Conditional Heteroscedasticity) model in 1982. Firstly, the model was generalised to Generalized Autoregressive Conditional Heteroscedasticity (Bollerslev, 1986), which was further modified to take into account various characteristics of financial data and as a result improving the volatility forecasting ability.

Many research papers show that accounting for additional non-linear relationships using artificial neural networks can improve forecasting ability for various time series (i.e.

BuHamra et al. 2003 or Zhang 2003). Similar conclusions can be drawn from papers which implement neural network models to forecast volatility of time series: Donaldson and Kamstra (1997), Si-ming et al. (2012), Pradeepkumar et al. (2017).

This chapter describes selected GARCH-type models as well as models based on artificial neural networks. For all analysed series of CDS spreads univariate models are estimated, including ones that account asymmetricity in response to positive and negative shocks EGARCH (Nelson, 1991) and GJR-GARCH (Glosten, Jagannathan and Runkle, 1993). Forecasts for all volatility models are compared with forecasts generated by the hybrid model based on recurrent neural networks, which is a modification of the model proposed by Kim and Won (2018). The comparison has been made for three forecast horizons: 1, 5 and 15 days with a Diebold-Mariano test (Diebold & Mariano, 1995). In order to validate robustness of results, the study analyses various measurements of volatility.

Verification of research hypotheses

1. **Assuming a volatile LGD (Loss Given Default) over time significantly influences estimations of PD (Probability of Default) and the fit to data of the CDS pricing model;**

The estimated reduced-form model to price sovereign CDS contracts on Polish bonds have achieved a superior fit between the model and the data (measured by R^2 and the Akaike Information Criterion). The method allowed for the identification of the PD values and expected LGD values for Poland, with both measures observed at different periods and with various expectation horizons. The superiority of the model was also validated for other European countries. For all analysed countries the model-implied LGD remained very low (below 10%), while PD varied over time reaching values close to unity during turmoil periods. Furthermore, the analysis showed that obtained values are highly correlated with outer measures of sovereign risk such as debt-to-gdp ratio or benchmark bond yields.

2. **Taking into account a non-linear relationship between SCDS and an underlying bond using Copula functions significantly improves risk estimates of the portfolio consisting of the bond and a CDS;**

Based on VaR calculations for pairs of sovereign bonds and corresponding CDS contracts for six countries, it has been shown that in some cases applying copula as a measure of dependence between assets can lead to statistically superior risk measures. The presented results show that taking into account the dependence in tails of distributions allows for the best fit for the relationship between analysed assets for all six countries. The VaR estimations show a statistically significant improvement in 1-day portfolio risk forecasts in three cases and the deterioration in only one case implying the superiority of the proposed method.

3. **Neural Network forecasting models significantly improve forecasts of the sovereign bonds volatility;**

The formal comparison of the forecasts generated by both a simple financial model and a hybrid model has shown that the hybrid model is superior to both the naive forecast and underlying simple models (GARCH-type models). The above is, however, true only for longer time horizons (five and 15 days). For very short time periods (1-day), the naive forecast remains the most effective in terms of the Root Mean Square Error. The results have been validated for alternative measures of volatility and are concluded to be robust. Using squared returns as a proxy for volatility, as well as extending the estimation window for the proposed volatility measure has led to very similar results.

Thesis contents

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Konrad Kostrzewa